High average power ultrafast lasers

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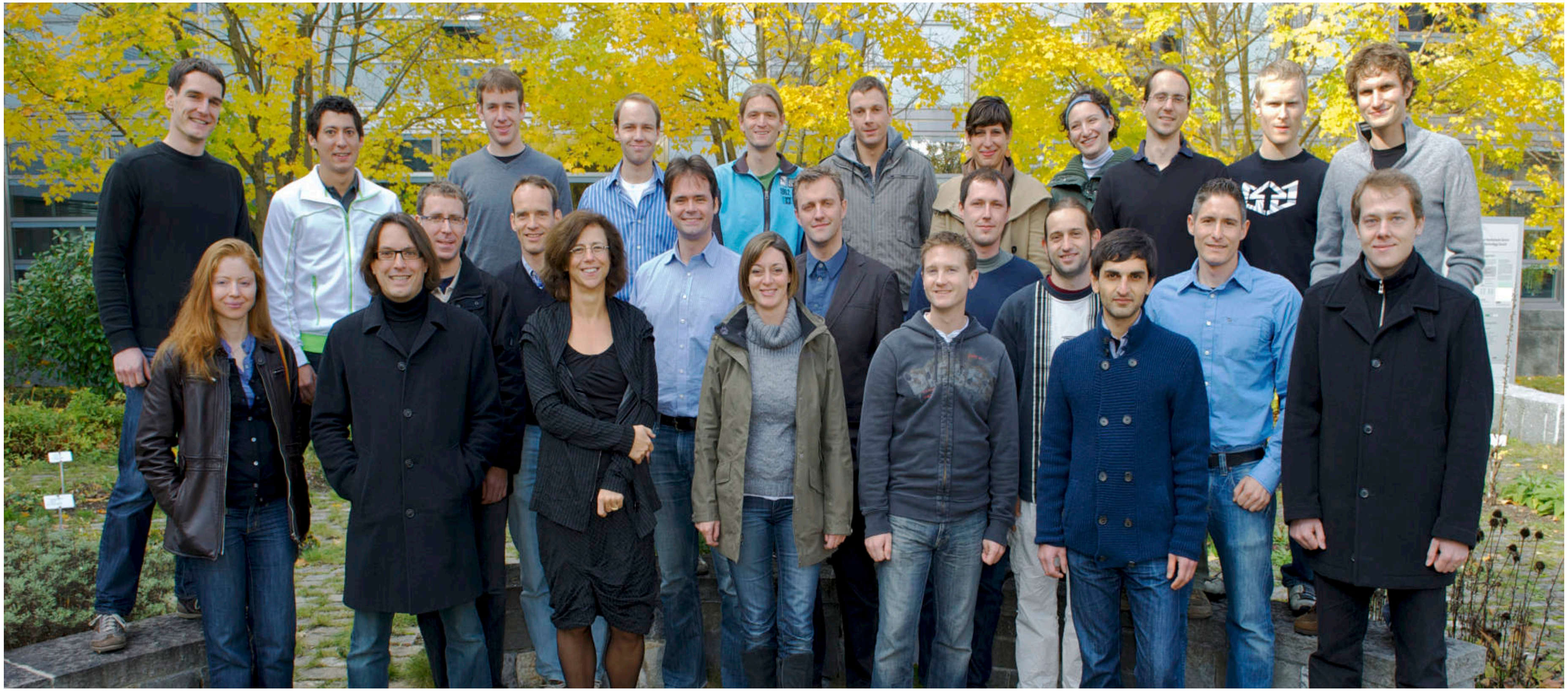
Neuchâtel, Switzerland

ETH – inspire – IWF Seminar, ETH Zurich
21. March, 2013

Ultrafast Laser Physics (ULP), Prof. Keller, ETH

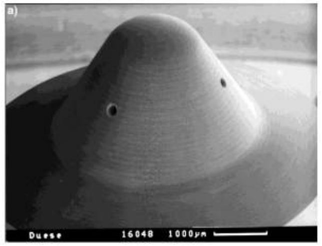


- Typically between 25 and 30 people (so far graduated 50 Ph.D. students)
- Two larger sub-groups with
  applied (ultrafast laser development) & fundamental (attosecond science) research
- Ultrafast laser development:
  - high average power (multi-100 W)
  - high pulse repetition rate for optical communication, interconnect, clocking
  - compact frequency combs (with novel ultrafast semiconductor lasers)

Applications

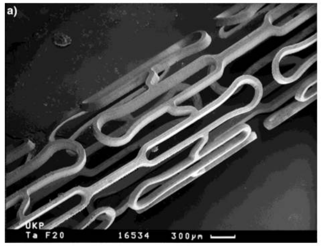
Industry

High-precision and high-speed micromachining



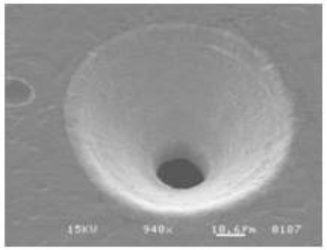
Fuel injection nozzles

Profeta et al., *Industrial Laser Solutions*, 2004



Stents

Nolte et al., *Adv. Eng. Mater.* 2, 2000



Inkjet nozzles

Liu et al., *Proc. SPIE*, Vol. 5713, 2005

Required

- High E_P > 10 µJ
- High P_{pk} > 10 MW

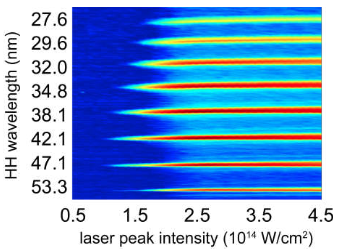
Wanted

- High f_{rep} (MHz)

➡ **High average power**
$$P_{av} = E_p \cdot f_{rep}$$

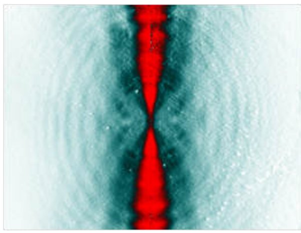
Science

Strong-field physics applications



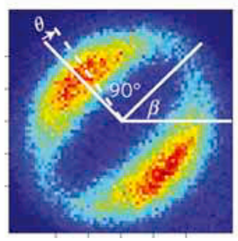
HHG

T. Auguste et al., *PRA* **80**, 033817 (2009)



Spectroscopy

T. Südmeyer et al., *Nat. Phot.* **2**, 599 (2008)



Attosecond science

A. Pfeiffer et al., *Nat. Phys.* **8**, 76 (2012)

Required

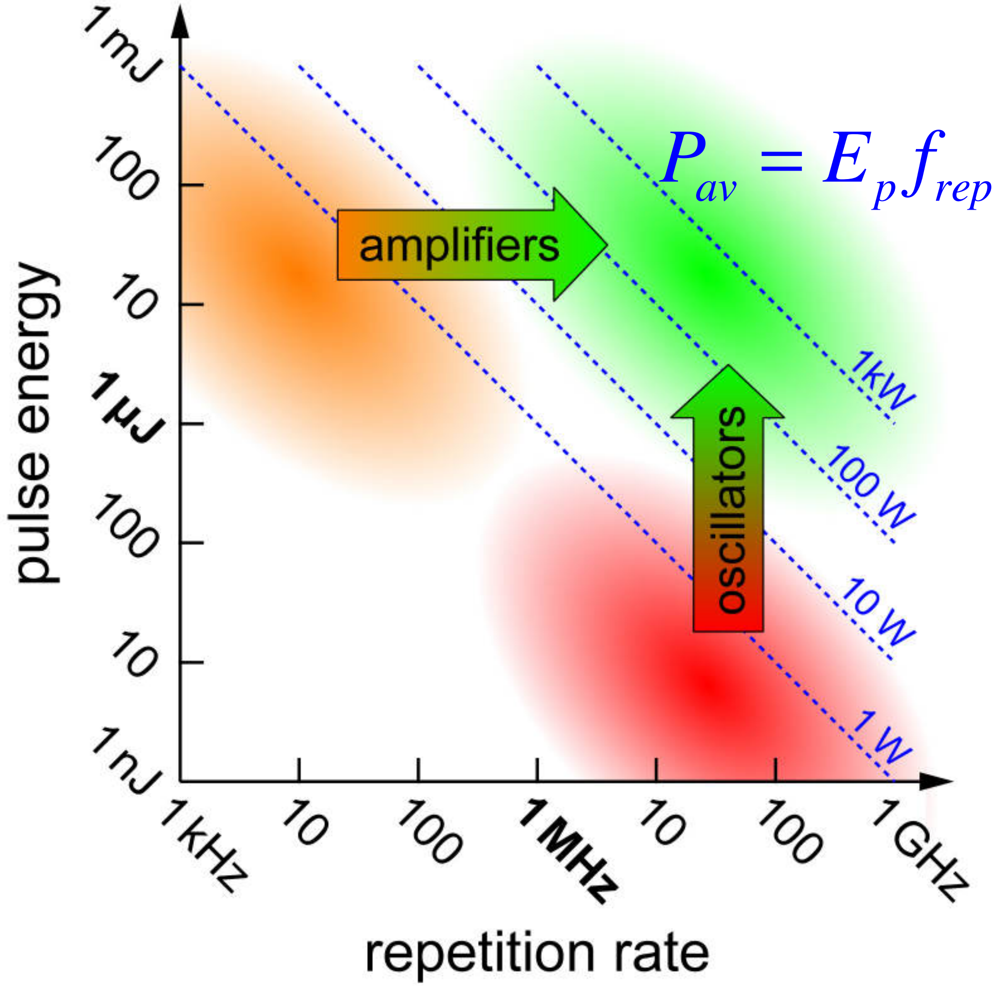
- High $I_{pk} > 10^{14}$ W/cm^2
- Short pulses τ_p < 100 fs

Wanted

- High f_{rep} (MHz)

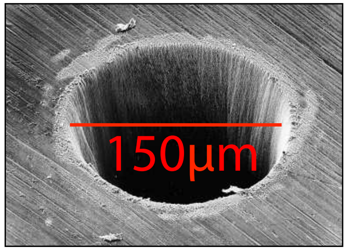
➡ **High average power**
$$P_{av} = E_p \cdot f_{rep}$$

High average power ultrafast solid-state lasers



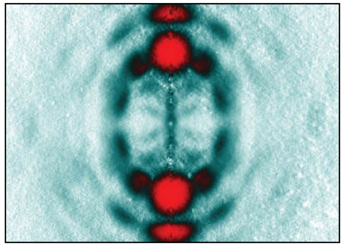
High energy and MHz

- **Industrial applications**
  - increase throughput,
  - reduce costs per item, …



B. N. Chichkov, et al.,
Appl. Phys. A **63**, 109 (1996)

- **Scientific applications**
  - reduce measurement time,
  - increase signal-to-noise,
  - MHz XUV sources, …



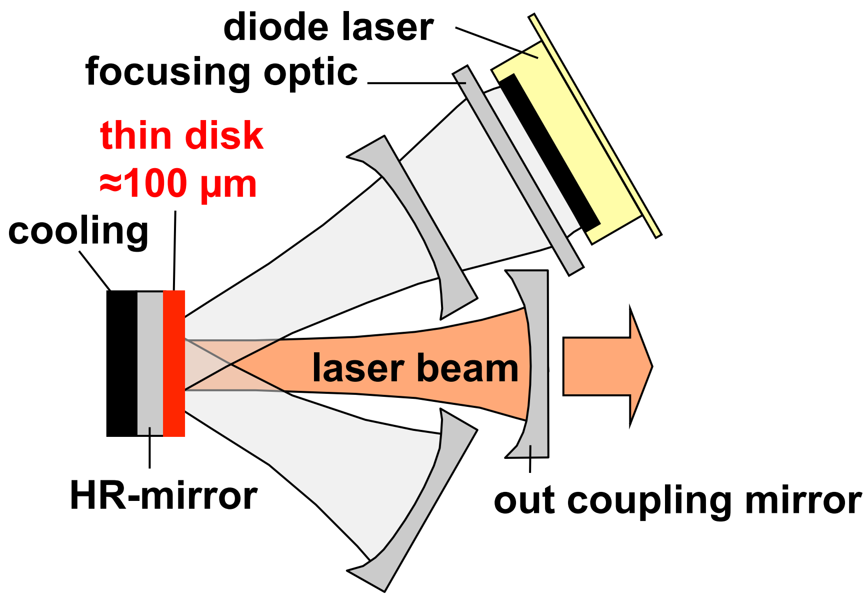
T. Südmeyer, et al., "Femtosecond laser oscillators for high-field science", *Nature Photonics* **2**, 559 (2008)

Power scaling: Advanced concepts for heat removal

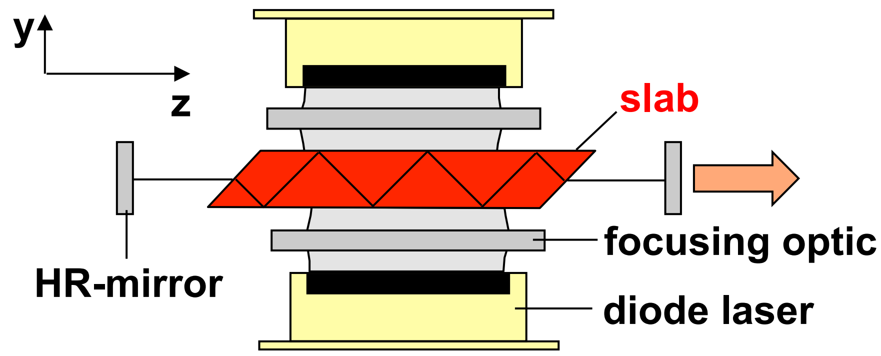
Heat removal: optimize surface to volume ratio

$$P_{av} = E_p f_{rep}$$

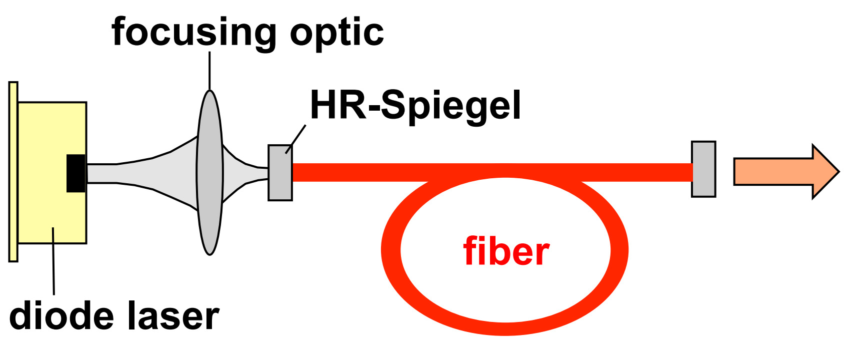
thin disk laser



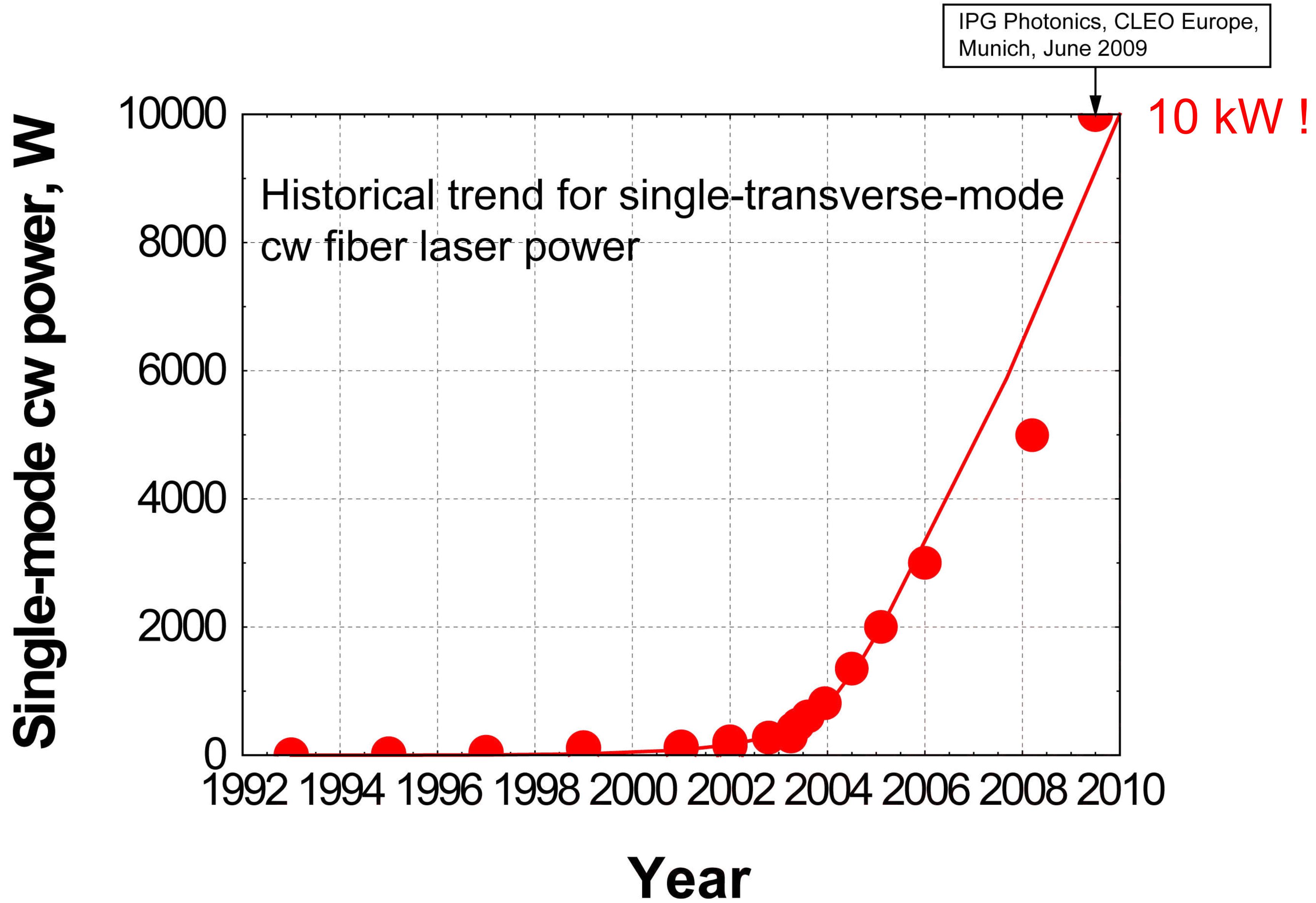
slab type laser



fiber laser



Fiber laser average power revolution



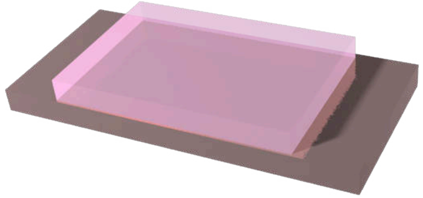
Courtesy of A. Galvanauskas, University of Michigan

High average power ultrafast sources

Key for high average power: heat removal

- Optimization of surface-to-volume ratio for efficient cooling

✓ Fiber

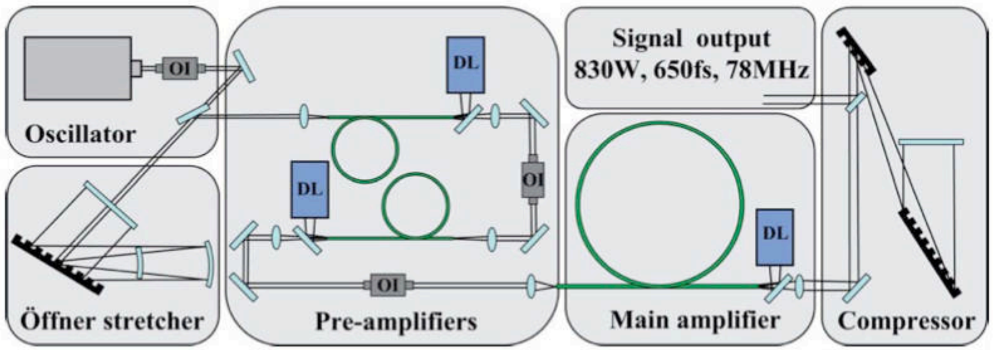


✓ Slab

✓ Thin disk

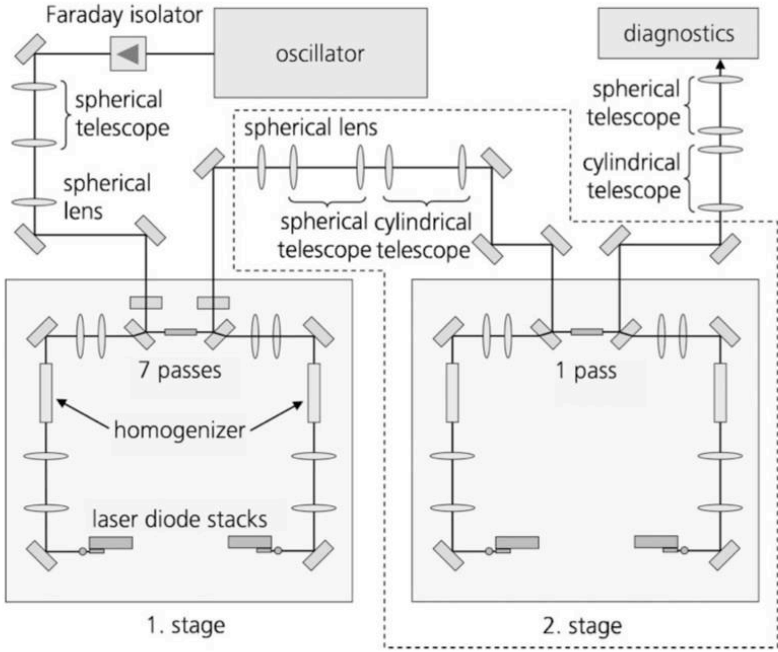
Key for ultrafast: reduce nonlinearities

- Operation at reduced peak intensity
- Reduced interaction volume



✓ Chirped pulse amplification (CPA) : 830 W, 640 fs

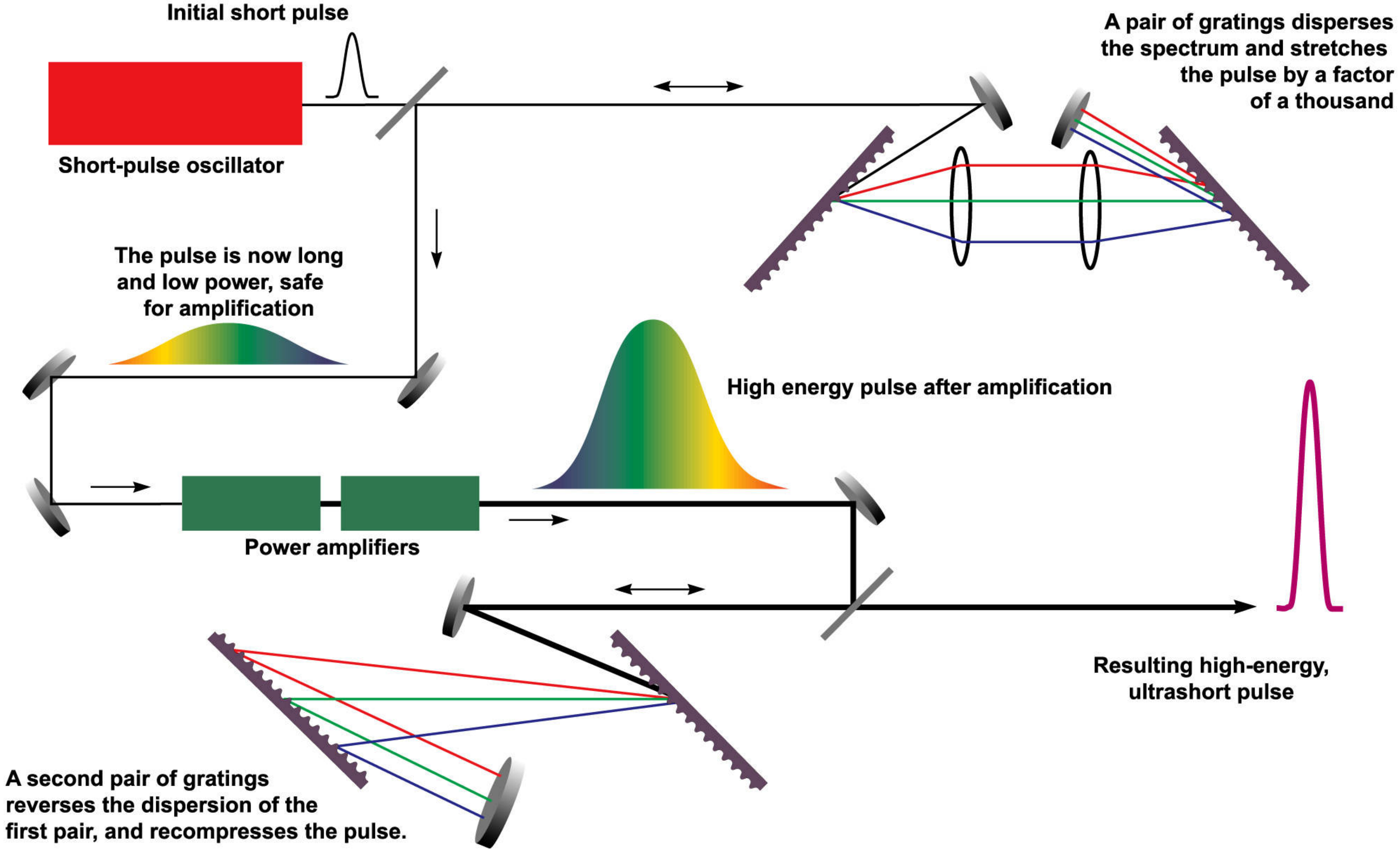
T. Eidam, et al., Optics Letters 35, 94-96 (2010)



✓ Innoslab : 1.1 kW, 615 fs

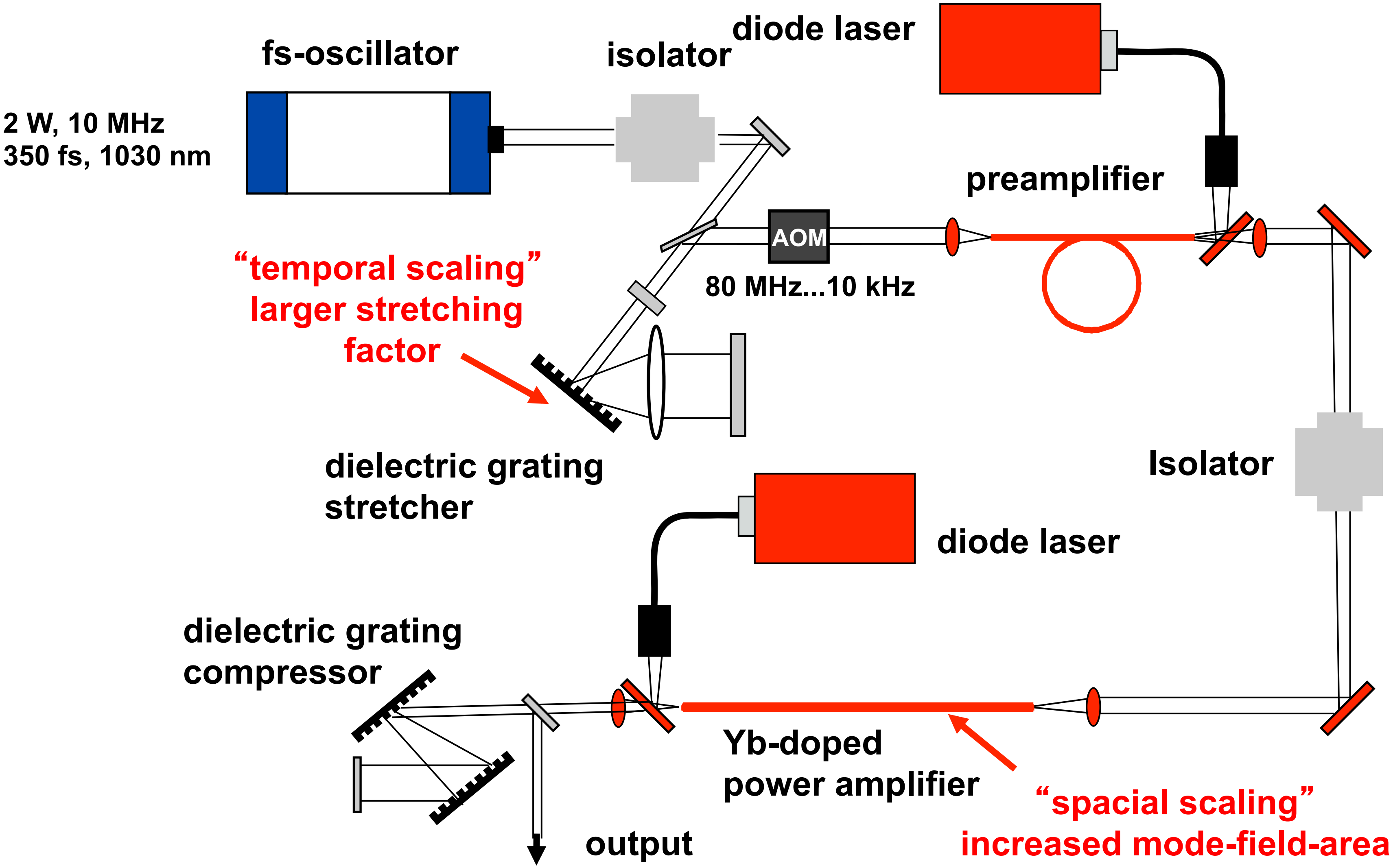
P. Russbueldt, et al., *Opt. Letters 35,* 4169-4171 (2010)

Chirped pulse amplification (CPA)

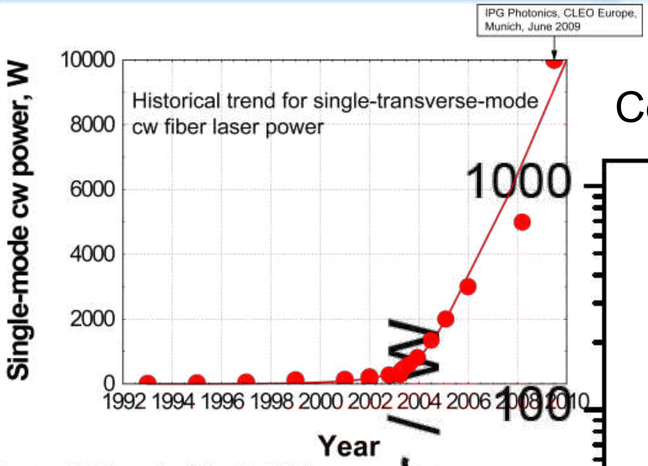


G. Mourou, 1987

Oscillator versus CPA-fiber amplifier

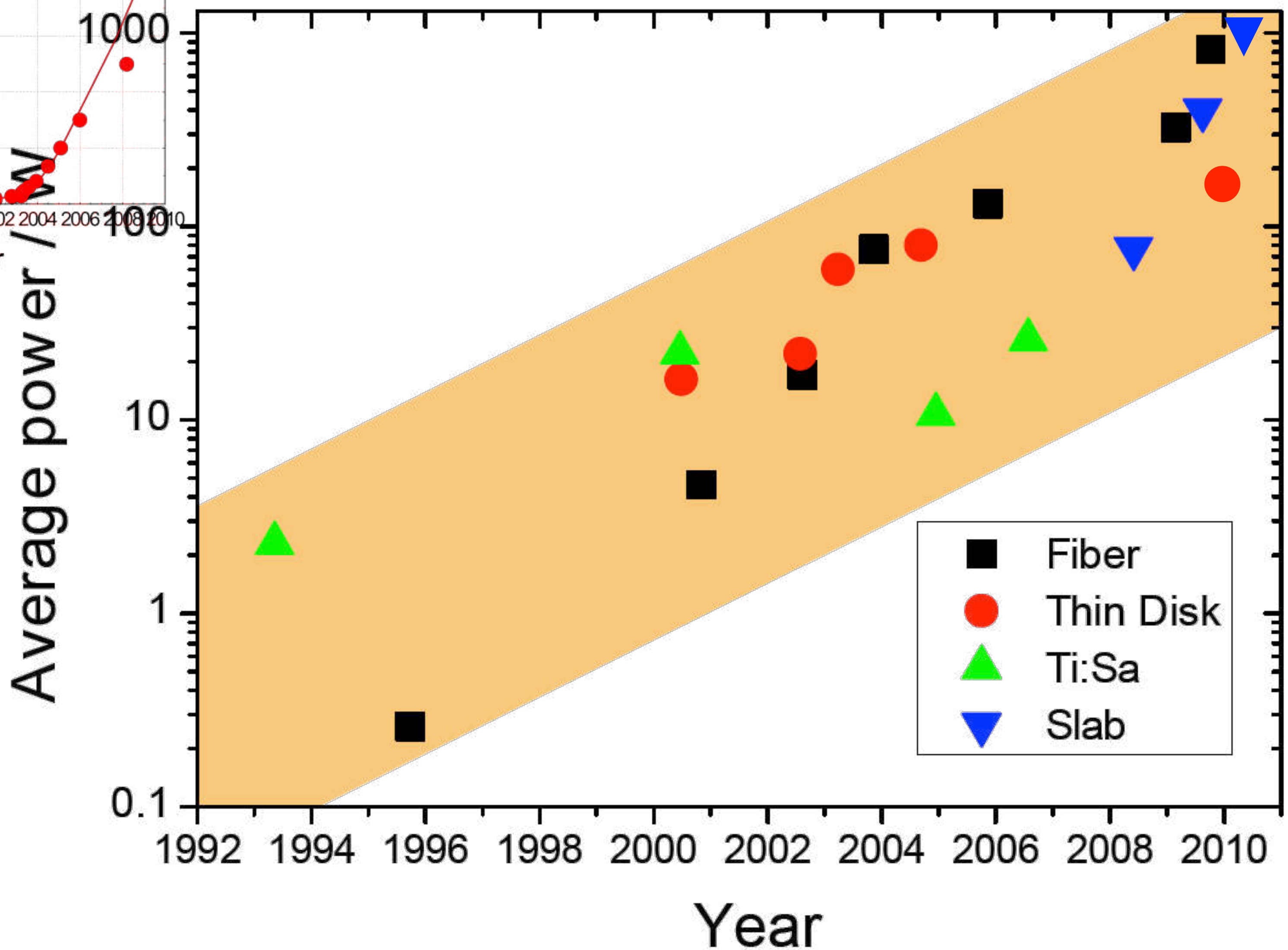


The femtosecond average power revolution



Courtesy of A. Galvanauskas, University of Michigan

Courtesy of A. Tünnermann: Systems with < 1ps pulse duration

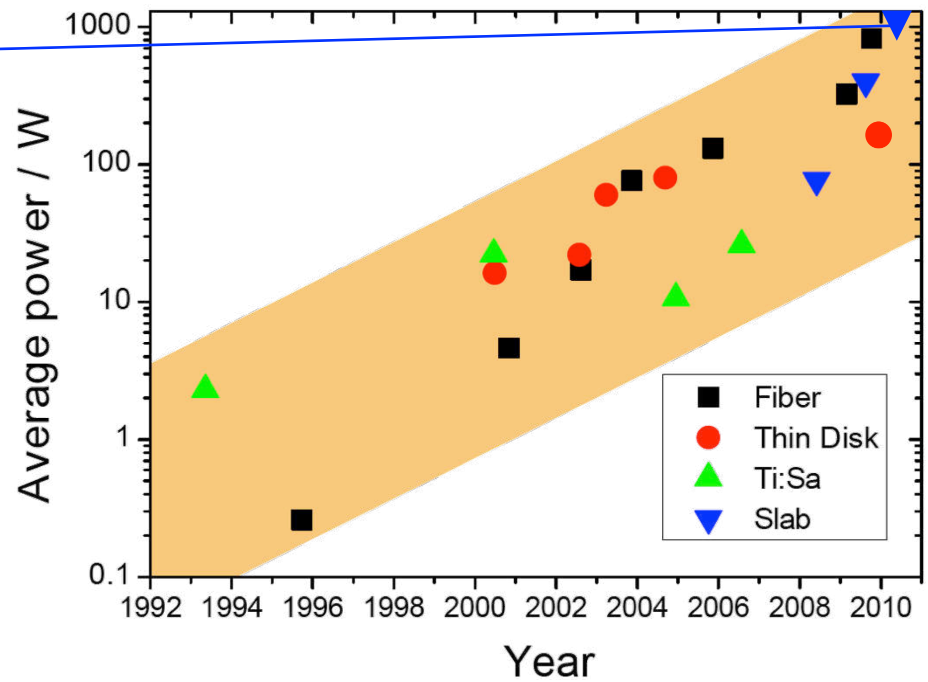


The femtosecond average power revolution

Femtosecond INNOSLAB amplifier (Poprawe):

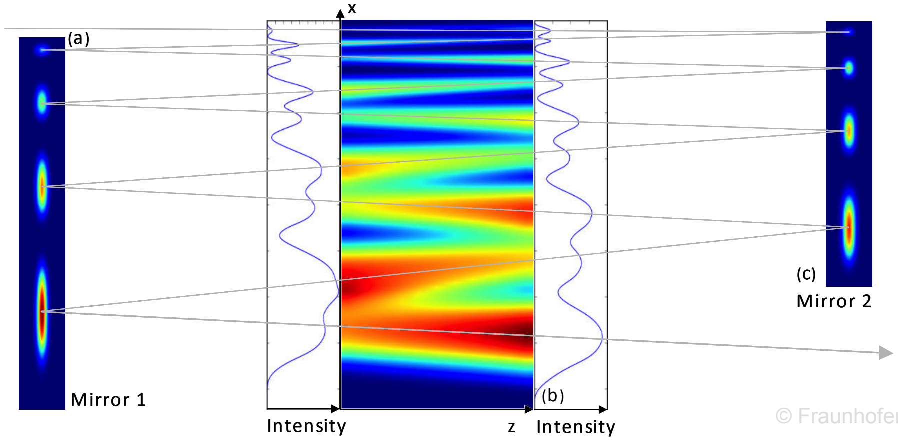
Yb:YAG dual stage Innoslab

Courtesy of A. Tünnermann: Systems with < 1ps pulse duration



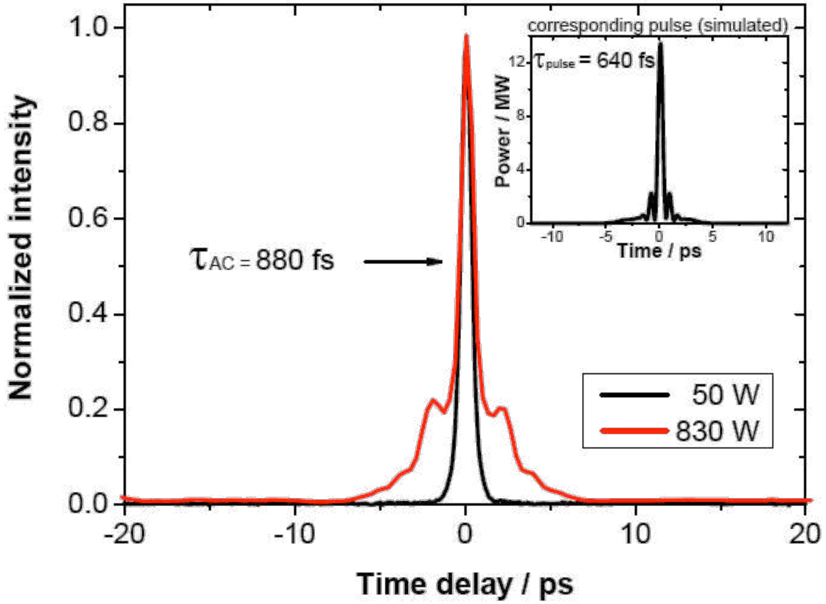
615 fs pulse duration
1.1 kW average power
90 MW peak power
20 MHz repetition rate
55 µJ pulse energy

Optics Lett. **35**, 4169, 2010

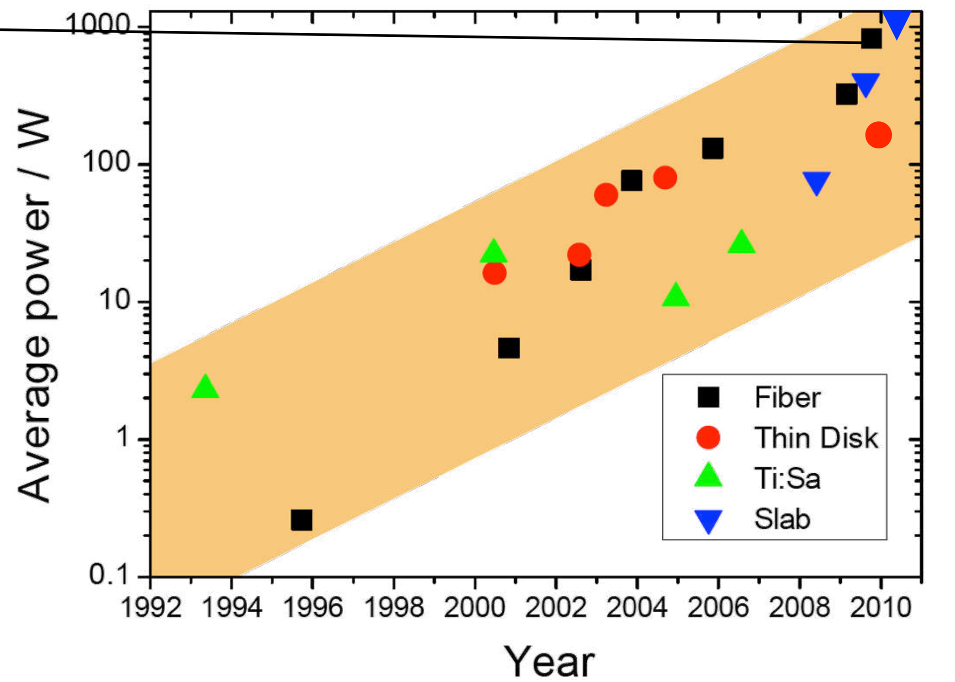


The femtosecond average power revolution

Femtosecond fiber amlifier system (Tünnermann):

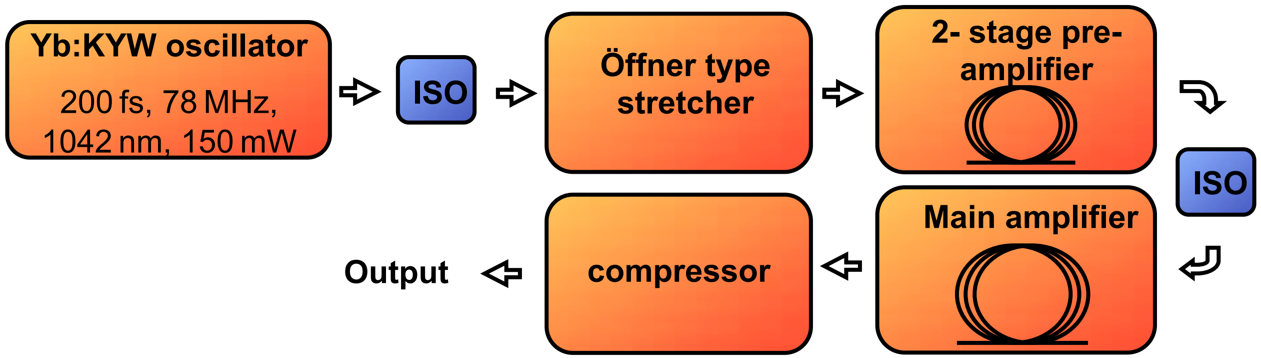


Courtesy of A. Tünnermann: Systems with < 1ps pulse duration



640 fs pulse duration
830 W average power
12 MW peak power
78 MHz repetition rate

Optics Lett. **35**, 94, 2010



Courtesy of A. Tünnermann

Fraunhofer IOF

Established industrial application - ps domain

advantage ps versus fs: less complexity

Nd:VAN
Ti:sapphire fs oscillator
80 MHz, nJ, 100 fs
10 ps

→ Stretcher fs to ns →

DUETTO amplifier

Nd:VAN
TiSA amplifier
μJ to mJ, Hz to kHz

← Q-switched green Pump laser
ns, mJ, Hz to kHz

↓ Compressor ns to fs

10 ps	<150fs
μJ to mJ	μJ to mJ
Hz to MHz	Hz to kHz

MOPA versus regenerative amps

- stability
- flexibility
- efficiency

Time-Bandwidth® Products

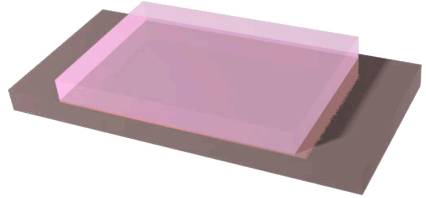
Example: Duetto
10 W, 200 μJ, 10 ps
(options for 50 W, green, UV)
Time-Bandwidth Products

High average power ultrafast sources

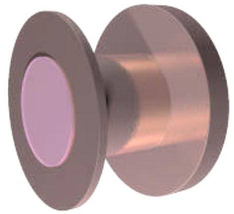
Key for high average power: heat removal

- Optimization of surface-to-volume ratio for efficient cooling

✓ Fiber



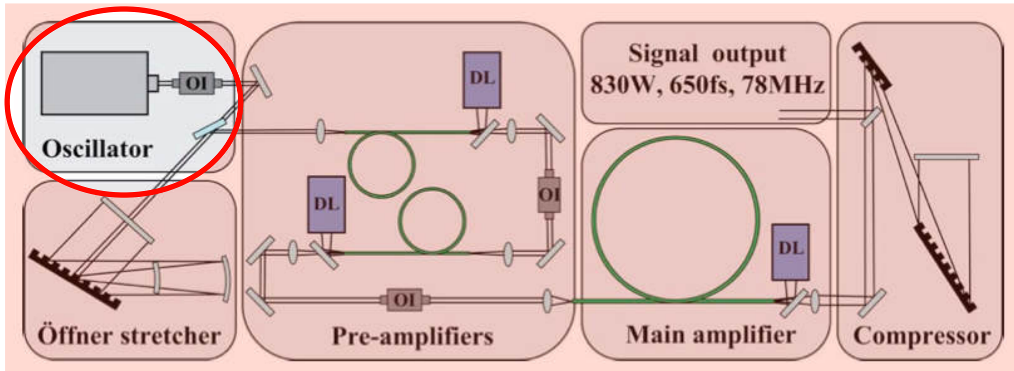
✓ Slab

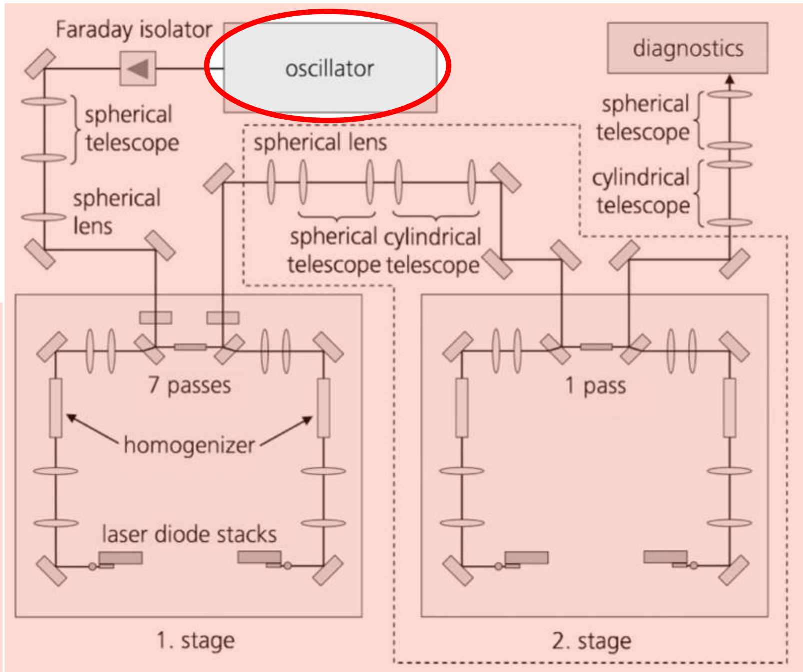


✓ Thin disk

Key for ultrafast: reduce nonlinearities

- Operation at reduced peak intensity
- Reduced interaction volume



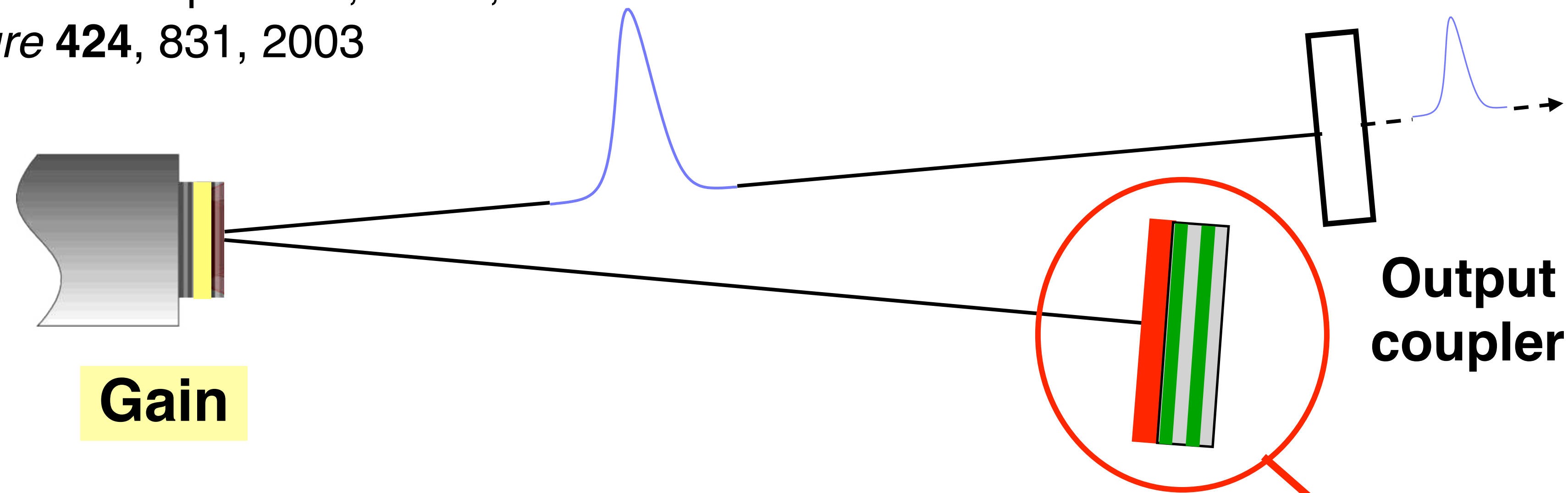


✓ Modelocked thin disk laser: high average power directly from the oscillator

SESAM technology – ultrafast lasers for industrial application

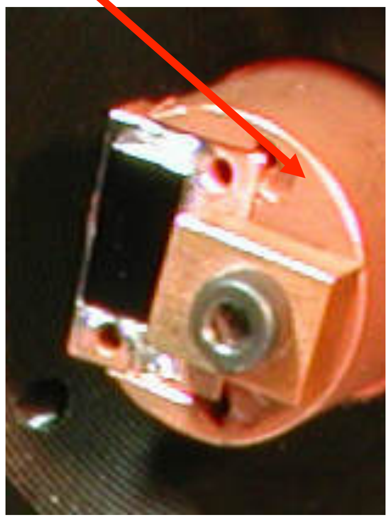
U. Keller et al. *Opt. Lett.* **17**, 505, 1992
IEEE JSTQE **2**, 435, 1996
Progress in Optics **46**, 1-115, 2004
Nature **424**, 831, 2003

SESAM solved Q-switching problem for diode-pumped solid-state lasers



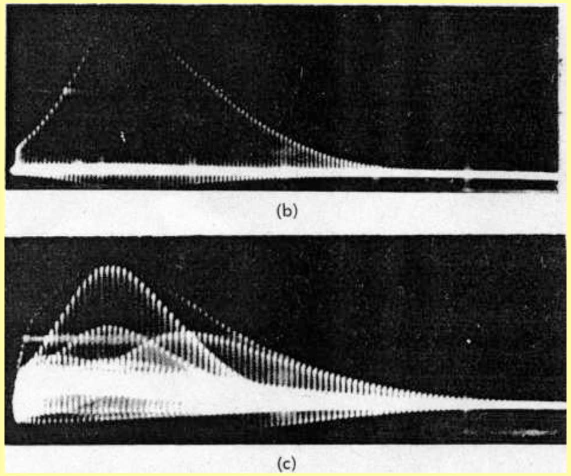
SESAM
SEmiconductor **S**aturable **A**bsorber **M**irror

self-starting, stable, and reliable modelocking of diode-pumped ultrafast solid-state lasers



Ultrashort pulse generation with modelocking

A. J. De Maria, D. A. Stetser, H. Heynau
Appl. Phys. Lett. **8**, 174, 1966



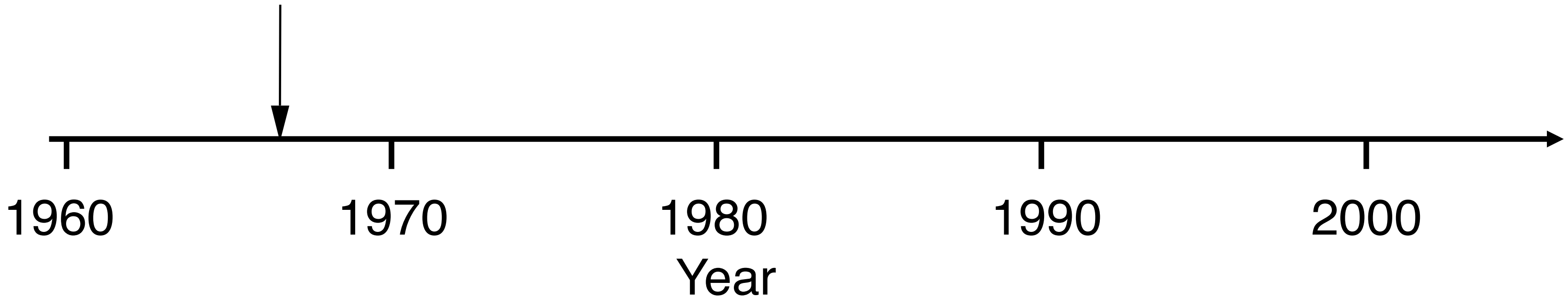
200 ns/div

50 ns/div

Nd:glass
first passively modelocked laser
Q-switched modelocked

Q-switching problem in passively modelocked solid-state lasers:

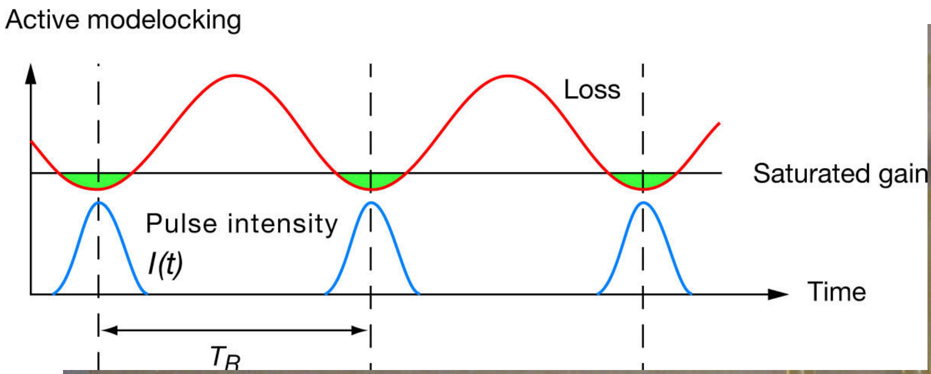
- active modelocking for solid-state lasers
- dye lasers solved the problem

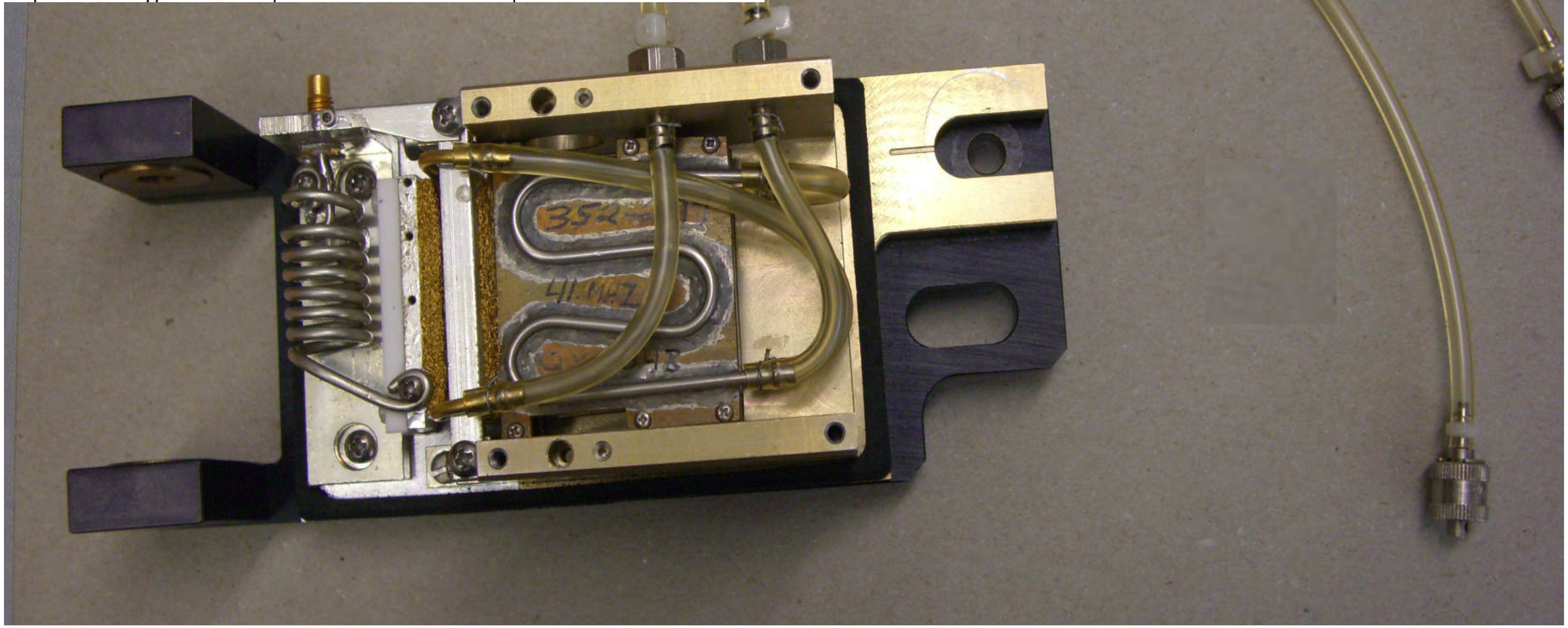


Flashlamp-pumped solid-state lasers

Diode-pumped solid-state lasers
(first demonstration 1963)

Active Modelocking

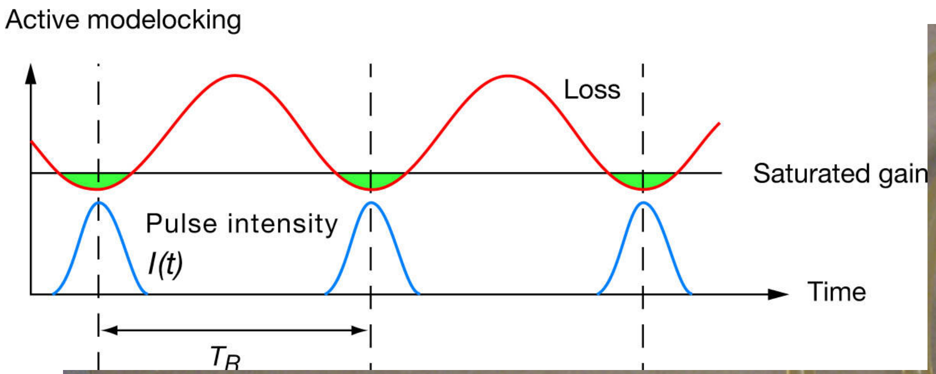


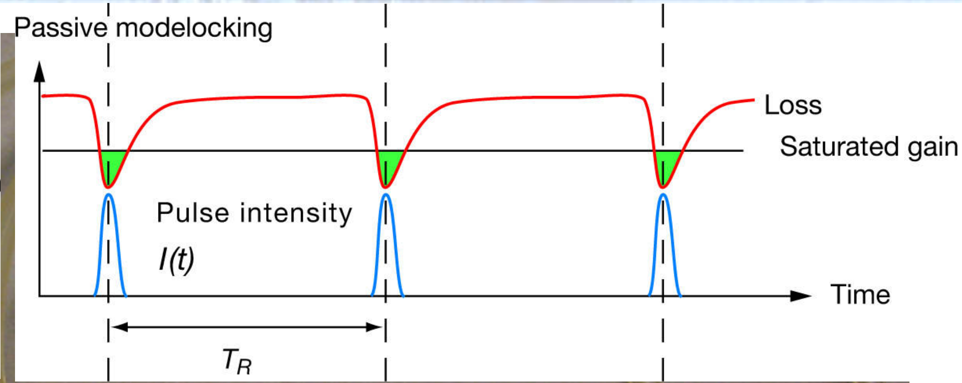


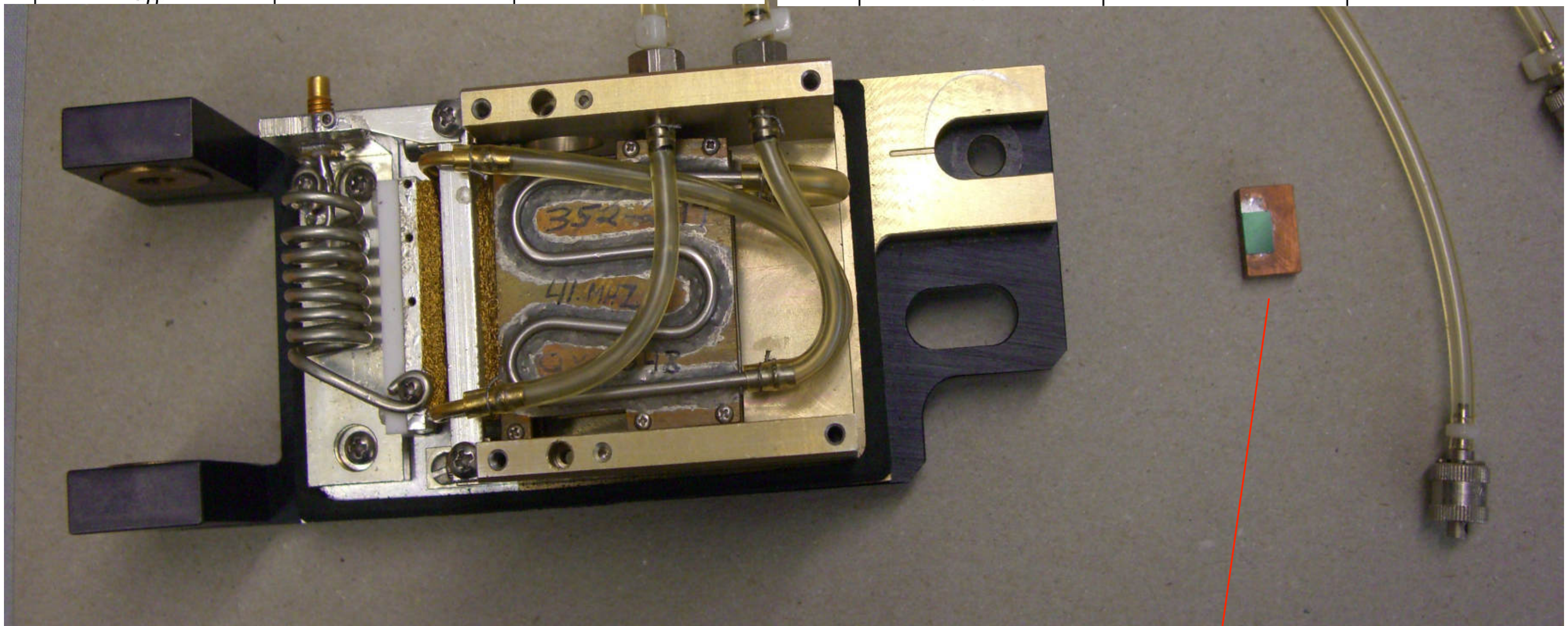
acousto-optic loss modulator

needs RF power and water cooling

Innovation: before and after





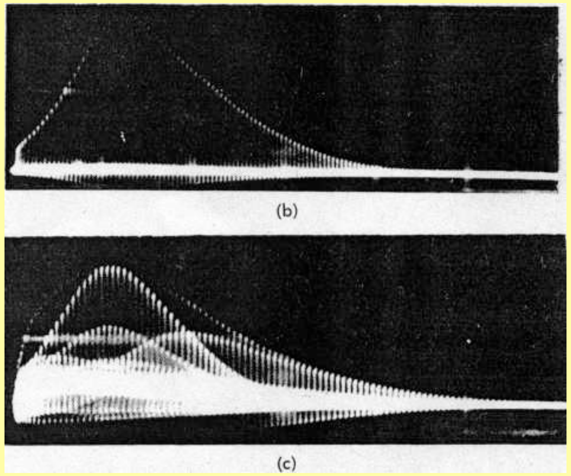


acousto-optic modelocker
needs RF power and water cooling

SESAM modelocker

20 years SESAM anniversary: 1992-2012

A. J. De Maria, D. A. Stetser, H. Heynau
Appl. Phys. Lett. **8**, 174, 1966



200 ns/div

50 ns/div

Nd:glass
first passively modelocked laser
Q-switched modelocked

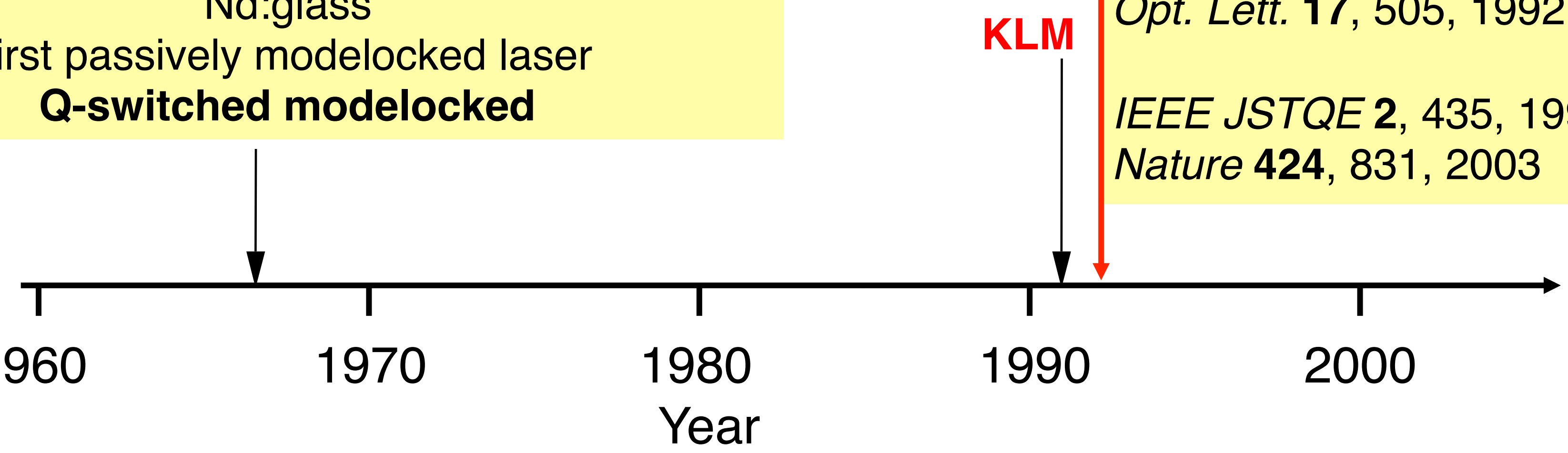
Q-switching instabilities continued to be a problem until 1992

SESAM

First passively modelocked (diode-pumped) solid-state laser without Q-switching

U. Keller et al.
Opt. Lett. **17**, 505, 1992

IEEE JSTQE **2**, 435, 1996
Nature **424**, 831, 2003



Flashlamp-pumped solid-state lasers

Diode-pumped solid-state lasers
(first demonstration 1963)

20 years of SESAM – looking back

Appl Phys B (2010) 100: 15–28
DOI 10.1007/s00340-010-4045-3

Appl. Phys. B **100**, 15-28, 2010

Applied Physics B
Lasers and Optics

Ultrafast solid-state laser oscillators: a success story for the last 20 years with no end in sight

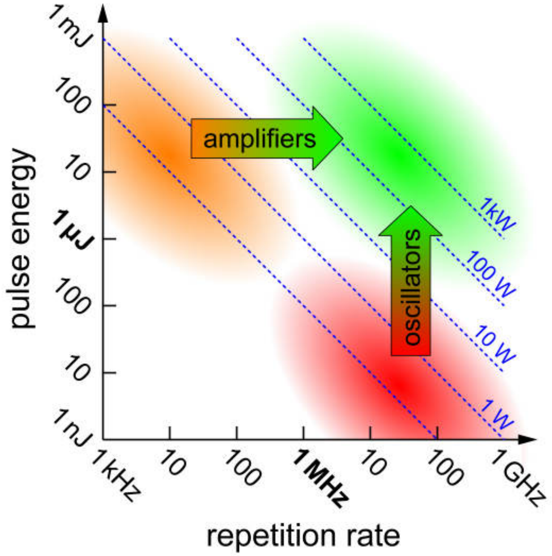
U. Keller

Received: 21 April 2010 / Published online: 13 May 2010

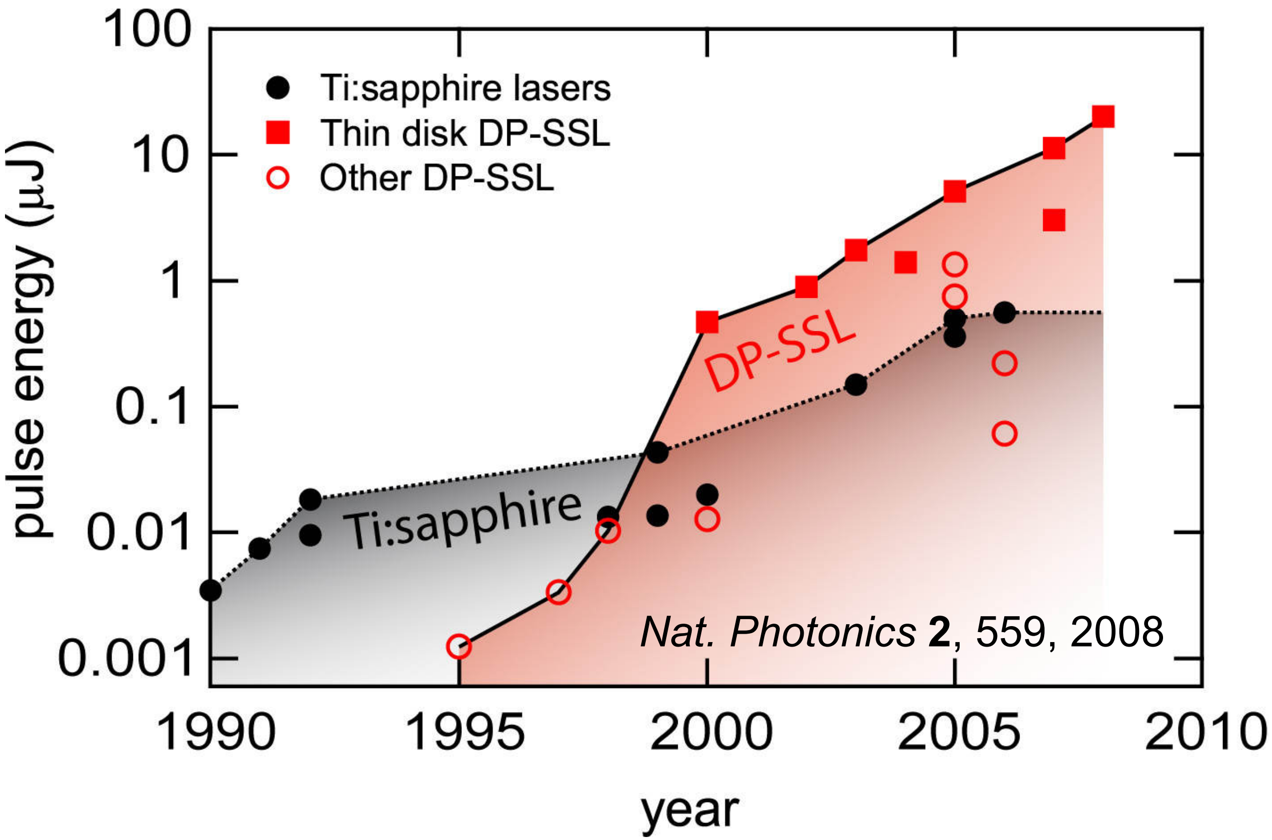
20 years of ultrafast solid-state lasers: invited paper

- Why was it assumed that diode-pumped solid-state lasers cannot be passively modelocked?
- How was the SESAM invented?
- State-of-the-art performance and future outlook.

High average power lasers



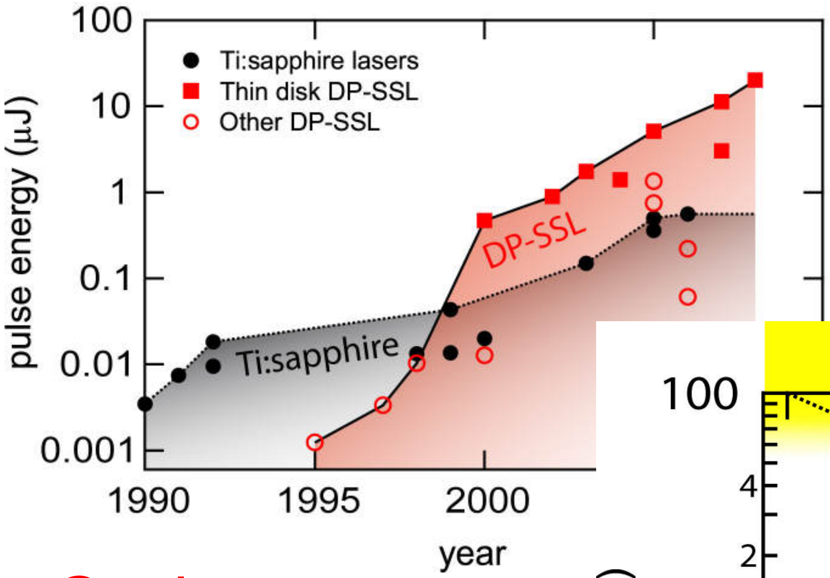
DP-SSL: diode-pumped solid-state lasers

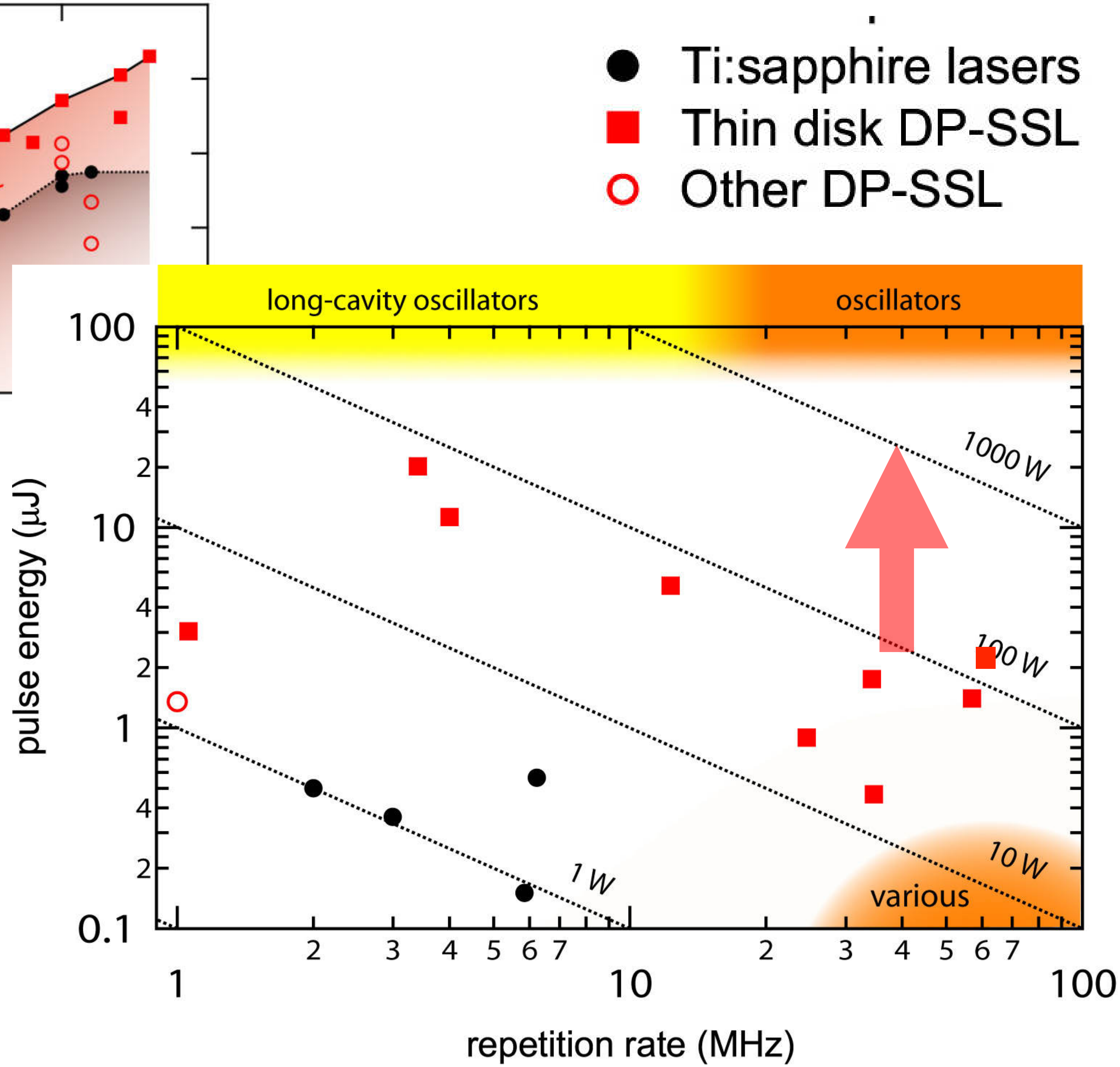


First time >10 µJ pulse energy from a SESAM modelocked Yb:YAG thin disk laser: *Opt. Express* **16**, 6397, 2008 and *CLEO Europe* June 2007

26 µJ with a multipass gain cavity and larger output coupling of 70% (Trumpf/Konstanz) *Opt. Express* **16**, 20530, 2008

High average power lasers - moving towards 100 µJ





Goal:

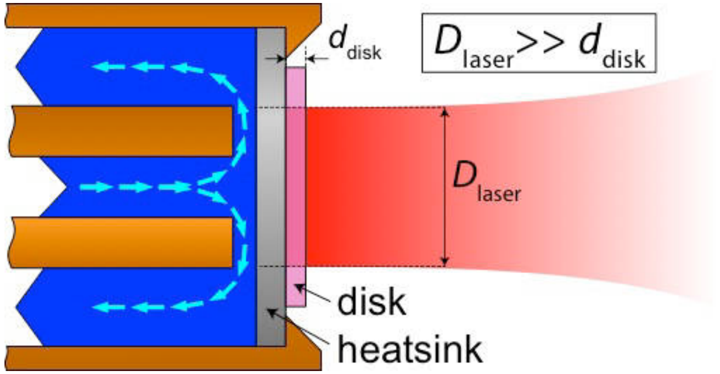
100 µJ

5 MHz

500 W average power

$$P_{av} = E_p f_{rep}$$

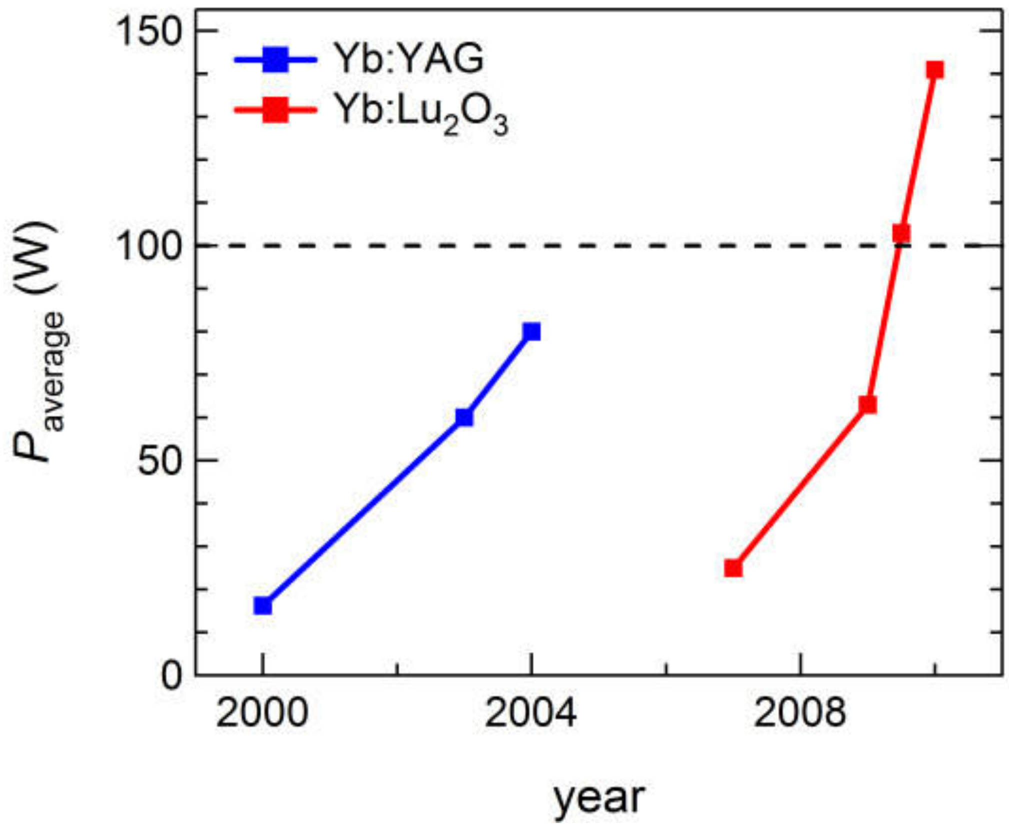
Thin disk laser



A. Giesen, et al., *Appl. Phys. B* **58**, 365 (1994)

- Efficient heat removal through back side
- Typical thickness ≈100 µm: 1D longitudinal heat flow: reduced thermal lensing
- **Power scalable by increase of mode diameter (constant intensity)**

Thin disk lasers are efficient



SESAM modelocked

η_{opt} = 42% (103 W)

η_{opt} = 40% (141 W)

C. R. E. Baer et al., *Optics Lett.* **35**, 2302, 2010

Yb-doped sesquioxide thin disk lasers
Prof. G. Huber, University of Hamburg

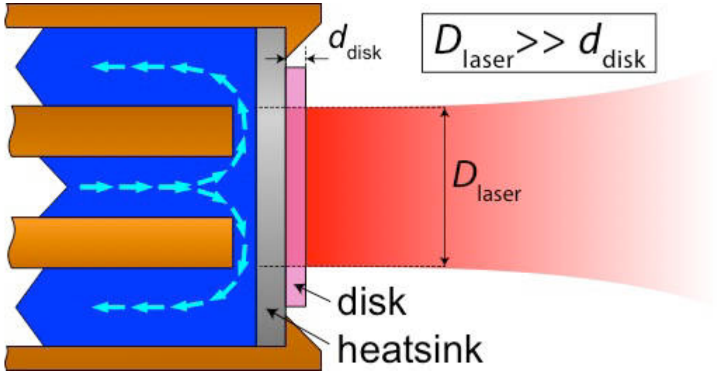
R. Peters et al., *Appl. Phys.* B 102, 509, 2011

cw performance

Gain material	P_{out} [W]	η_{opt} [%]	η_{slope} [%]
Yb:Lu_2O_3	301	73	85
Yb:Sc_2O_3	264	70	80
Yb:$LuScO_3$	250	69	81

Modelocked Thin Disk Lasers

Thin disk laser



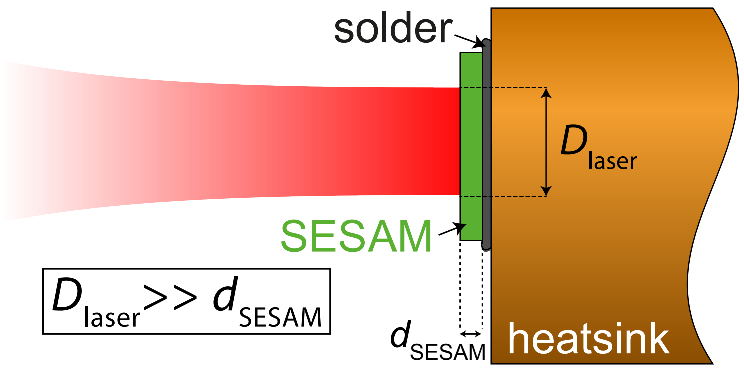
A. Giesen, et al., *Appl. Phys. B* **58**, 365 (1994)

- Efficient heat removal through back side
- Typical thickness ≈100 µm: 1D longitudinal heat flow: reduced thermal lensing
- **Power scalable by increase of mode diameter (constant intensity)**

+

SESAM

Semiconductor saturable absorber mirror

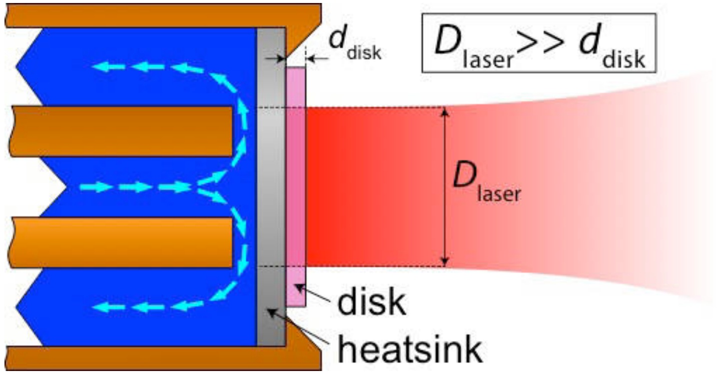


U. Keller, et al., *IEEE J. Sel. Top. Quant.* **2**, 435 (1996)

- Widely tunable absorber parameters
- 1D longitudinal heat flow: reduced thermal lensing
- High damage thresholds (>100 mJ/cm^2) for optimized designs
- **Power scalable by increase of mode diameter (constant saturation)**

Modelocked Thin Disk Lasers

Thin disk laser

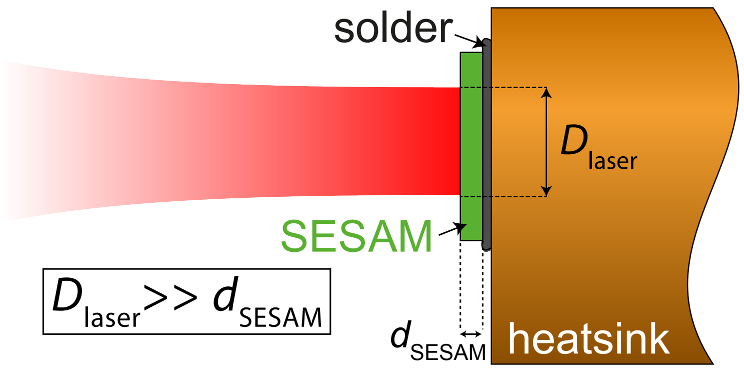


A. Giesen, et al., *Appl. Phys. B* **58**, 365 (1994)

+

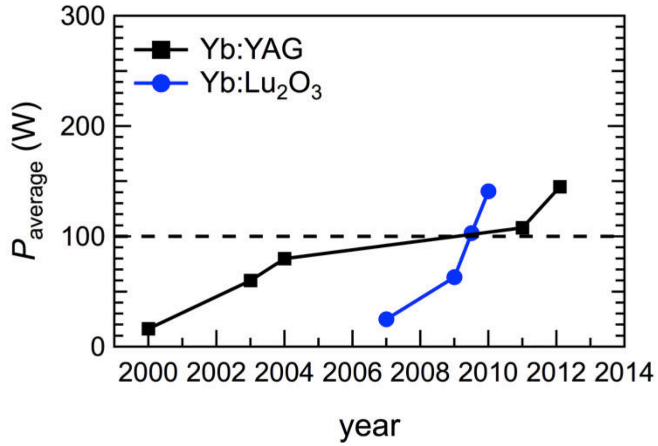
SESAM

Semiconductor saturable absorber mirror

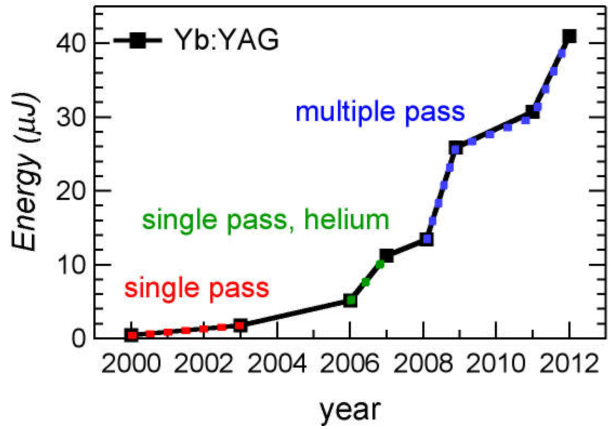


U. Keller, et al., *IEEE J. Sel. Top. Quant.* **2**, 435 (1996)

Highest average powers and highest energies of any ultrafast oscillator technology



***P_{av}* > 140 W** [#1, #2]

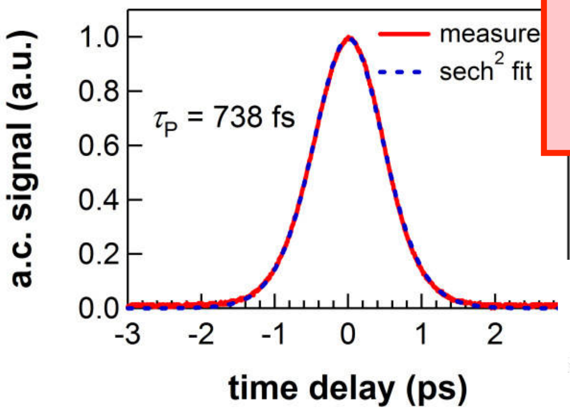


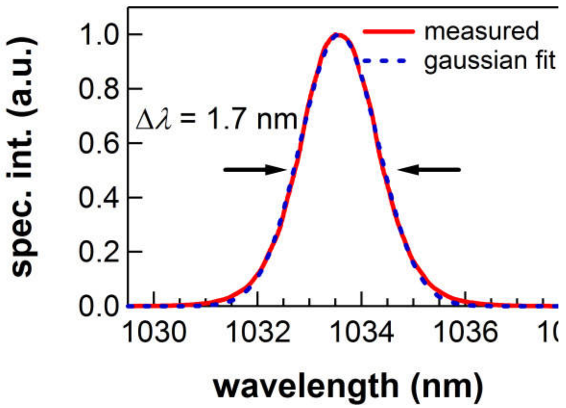
***E_p* > 40 μJ** [#2]

[#1] C. R. E. Baer, et al., *Optics Letters* ***35**, 2302-2304* (2010)
[#2] D. Bauer et al., *Optics Express* ***20**, 9698-9704* (2012)

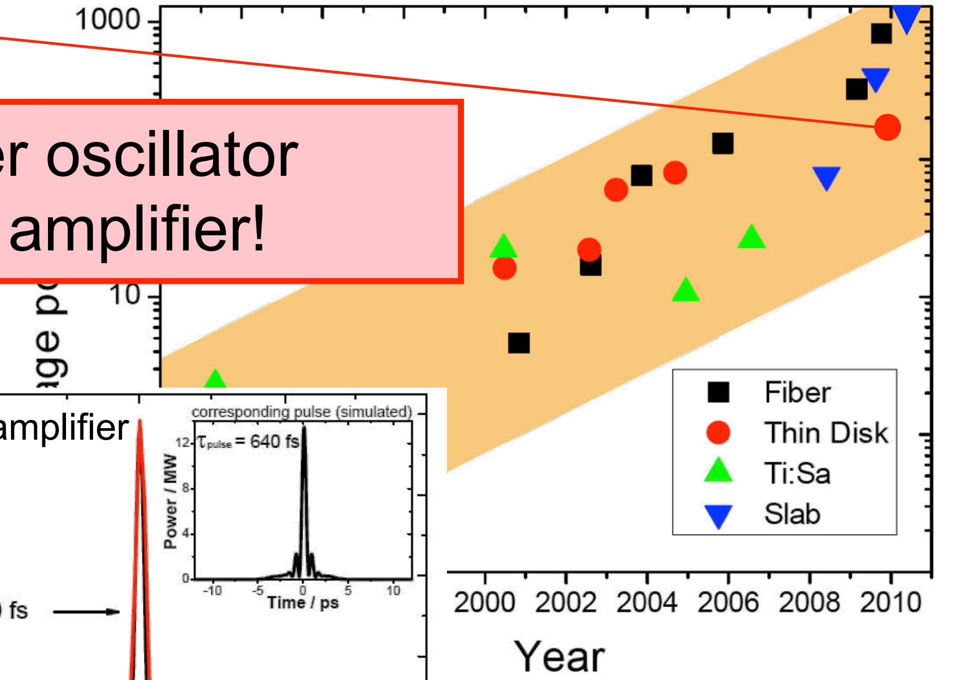
The femtosecond average power revolution

Femtosecond SESAM modelocked $Yb:Lu_2O_3$ thin disk laser

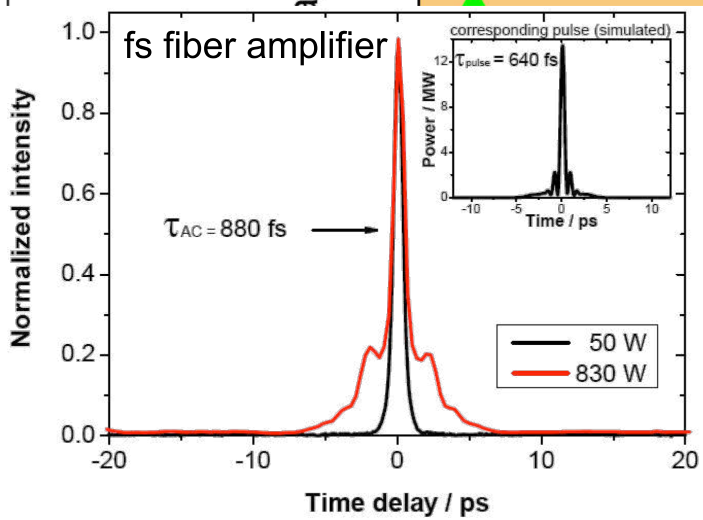


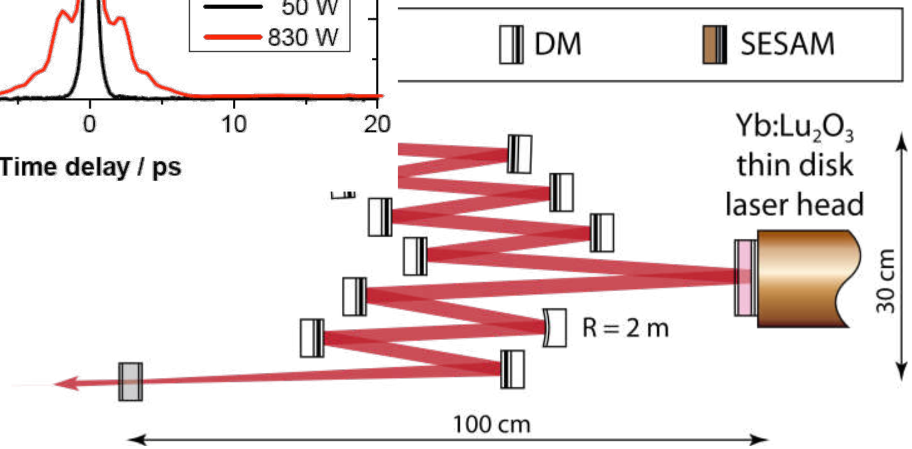


Courtesy of A. Tünnermann: Systems with < 1ps pulse duration



laser oscillator
no amplifier!





738 fs pulse duration

141 W average power

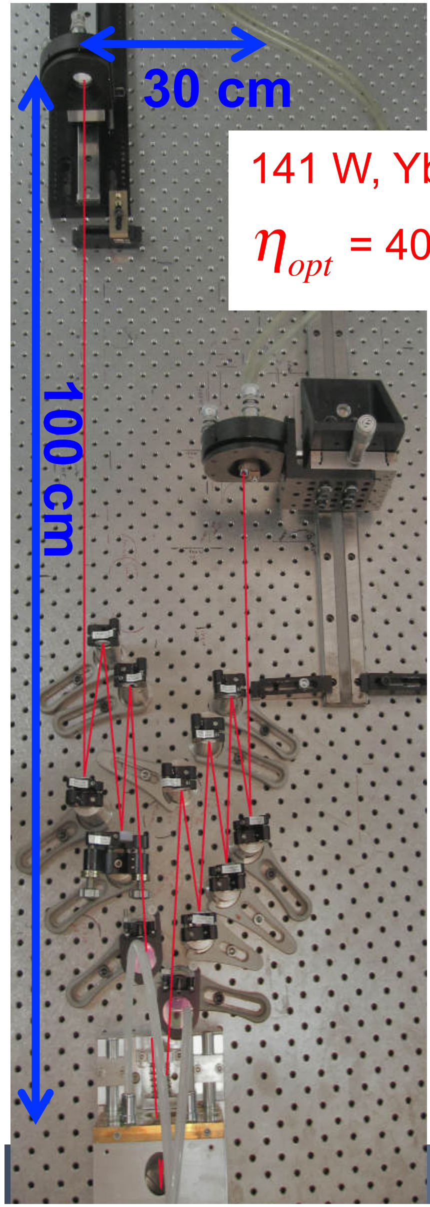
2.8 MW peak power

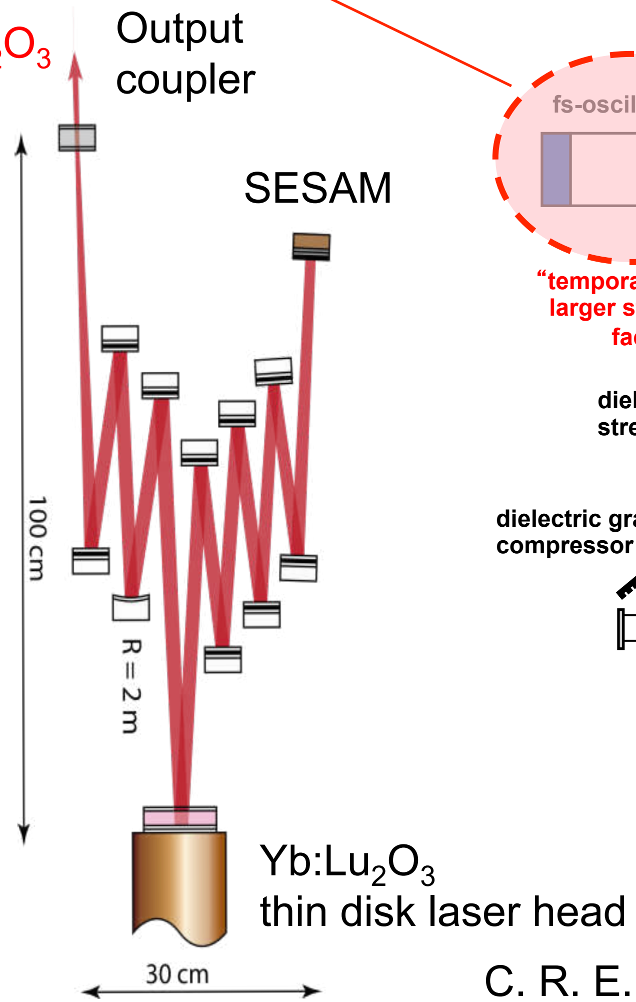
60 MHz repetition rate

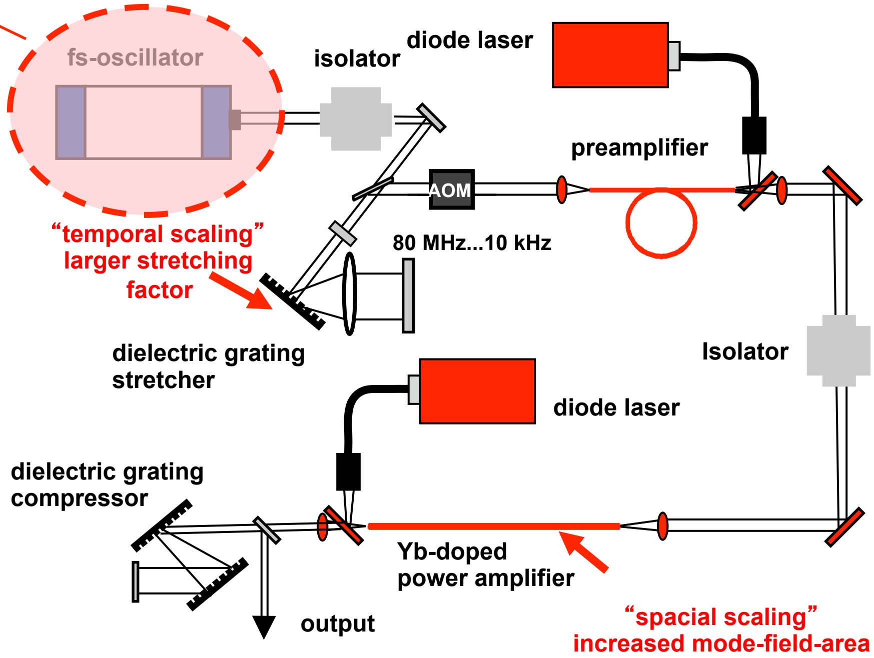
Oscillator versus CPA-fiber amplifier

High power fs oscillator: same complexity as low power fs oscillator

femtosecond CPA-fiber amplifier always needs a fs oscillator



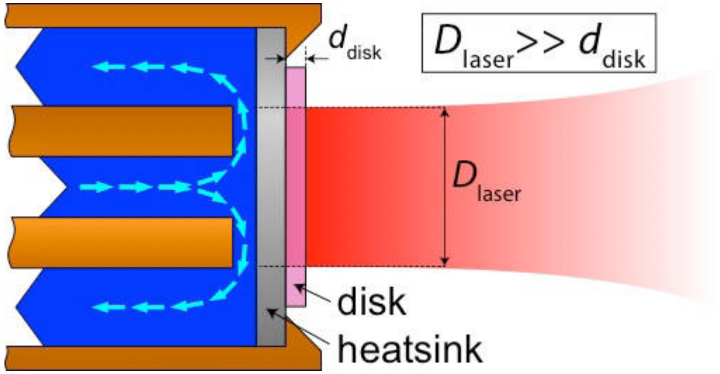




C. R. E. Baer et al., *Optics Lett.* **35**, 2302, 2010

Modelocked Thin Disk Lasers

Thin disk laser

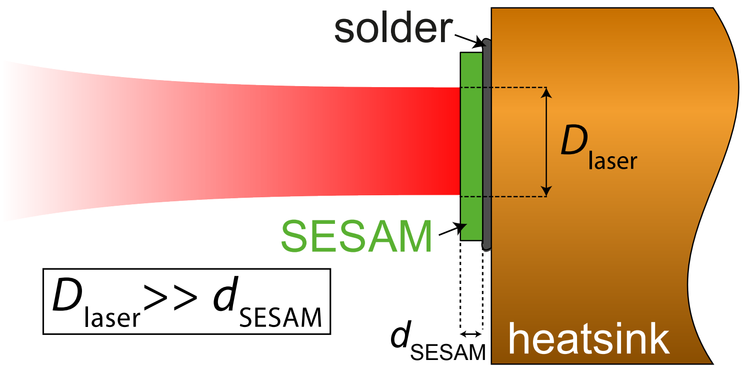


A. Giesen, et al., *Appl. Phys. B* **58**, 365 (1994)

+

SESAM

Semiconductor saturable absorber mirror



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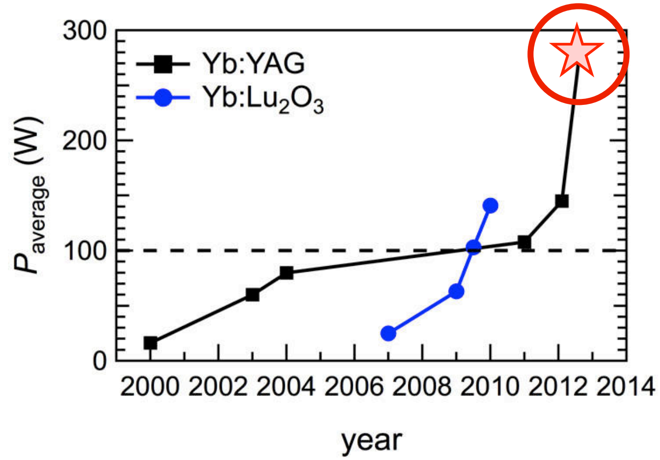
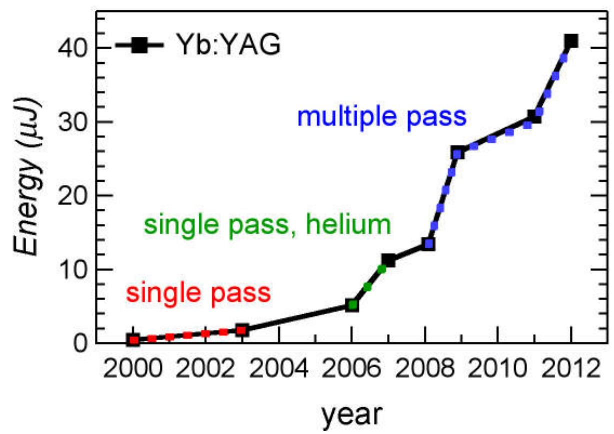


Here:

P_{av} = 275 W

τ_p = 583 fs

f_{rep} = 16.3 MHz



$E_p > 40\ \mu J$ [#2]

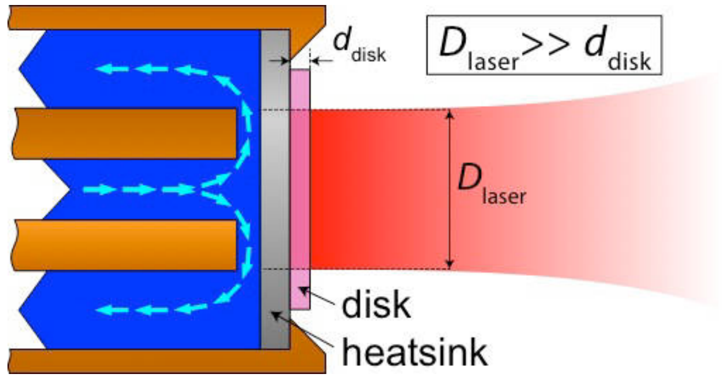
[#1] C. R. E. Baer, et al., *Optics Letters* **35**, *2302-2304* (2010)

[#2] D. Bauer et al., *Optics Express* **20**, *9698-9704* (2012)

High-power modelocking: challenges

TEM_{00} operation at high average power [#1]

- efficient heat removal:
  - → material properties: thermo-mechanical and spectroscopic properties
  - → disk quality: thickness, diameter
  - → contacting
- suitable cavity design



C. R. E. Baer, et al., *Optics Express* ***20****, 7054-7065* (2012)

Yb:YAG: the standard thin disk material

- large disks on diamond with excellent quality commercially available
- 500 W fundamental transverse mode demonstrated [#1]

[#1] A.Killi, et al., *Proceedings of the SPIE, Volume 7193,* 2009

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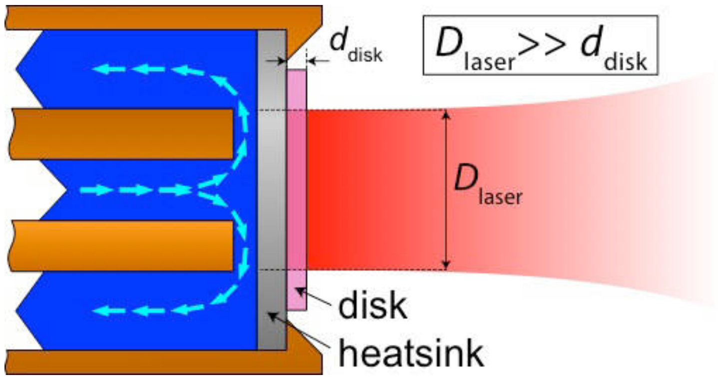
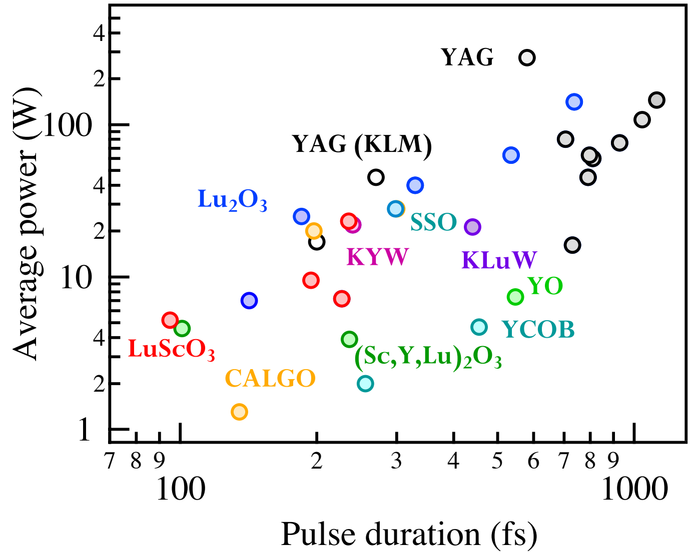


C. R. E. Baer, et al., *Optics Express* ***20***, *7054-7065* (2012)

Yb:YAG: the standard thin disk material

- large disks on diamond with excellent quality commercially available
- 500 W fundamental transverse mode demonstrated [#1]

Other materials with promising properties are currently being investigated



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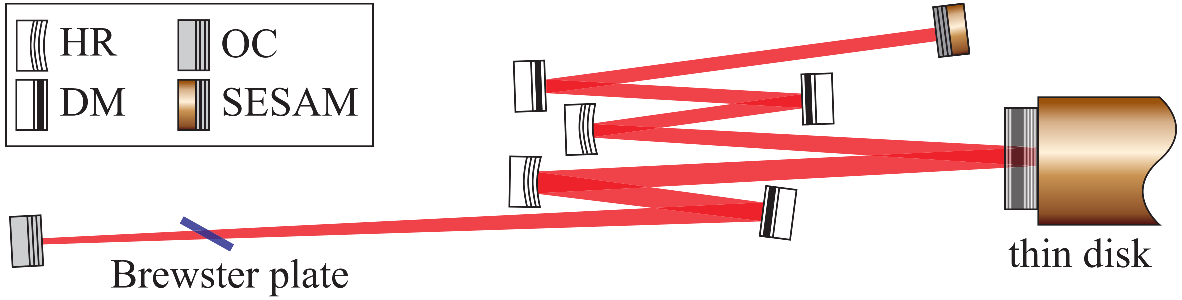
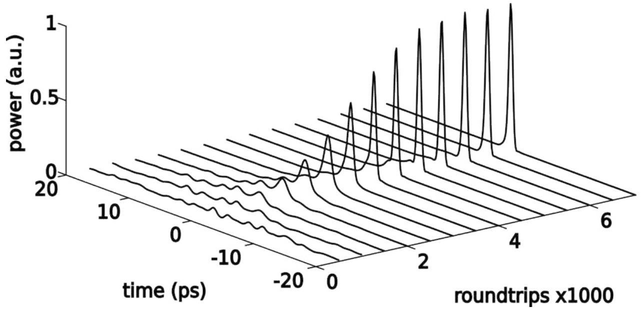
High-power modelocking: challenges

Pulse formation at high peak power:

Soliton modelocking: balance SPM and GDD





→ avoid modelocking instabilities from **excessive nonlinearities**

F. X. Kärtner and U. Keller, Opt. Lett. **20(1),** 16–18 (1995)

R. Paschotta and U. Keller, Appl. Phys. B **73(7),** 653–662 (2001)

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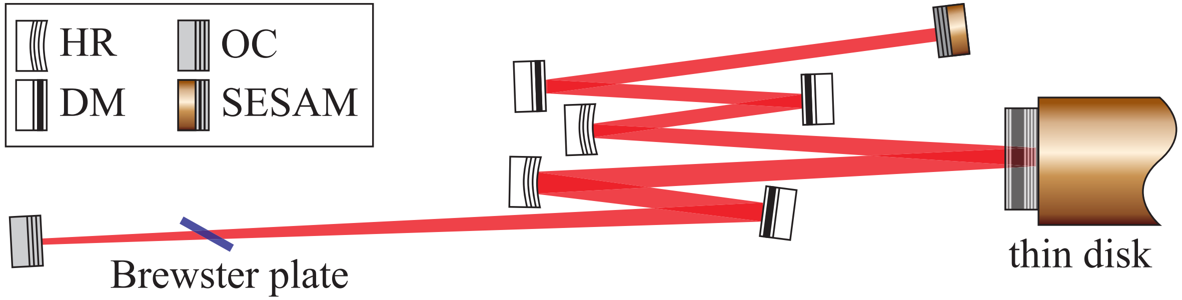

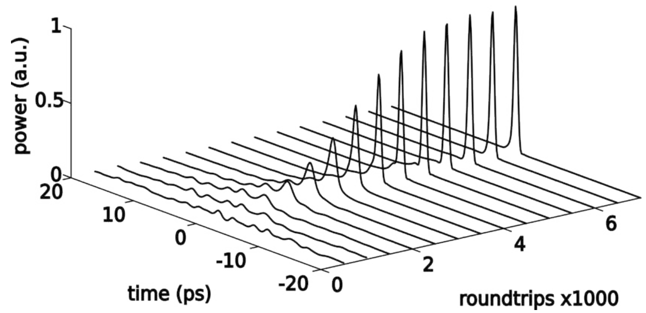

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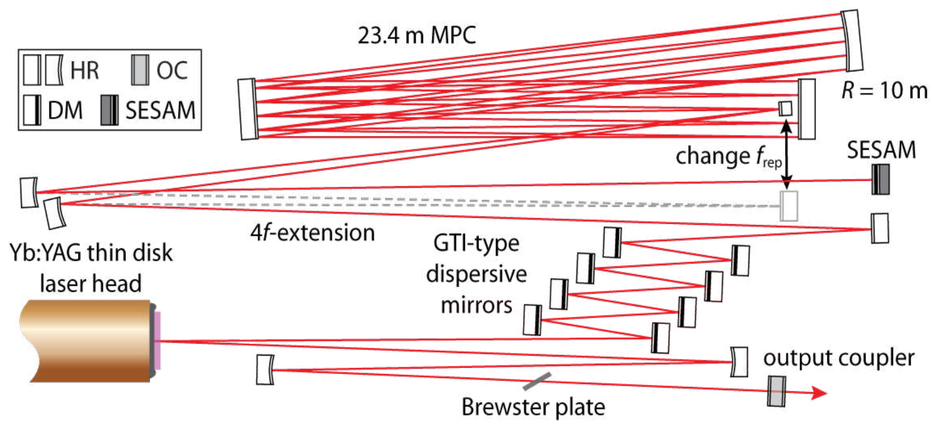
Helium flooding   45 W, 11 µJ, 790 fs



S. Marchese, et al., *Optics Express* ***16****, 6397-6409* (2008)

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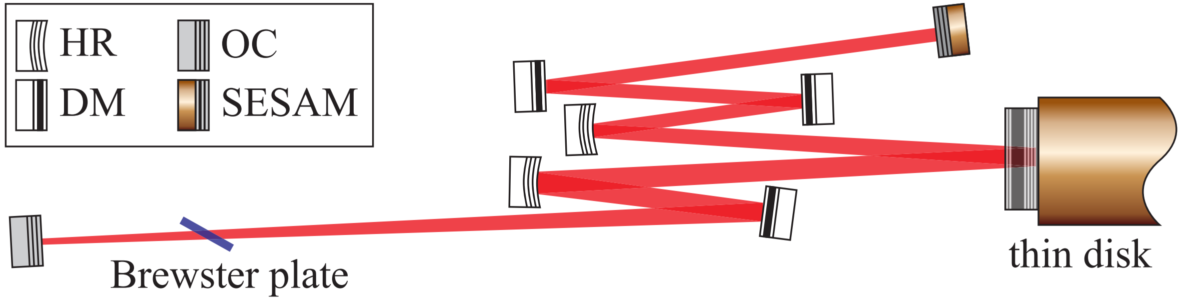

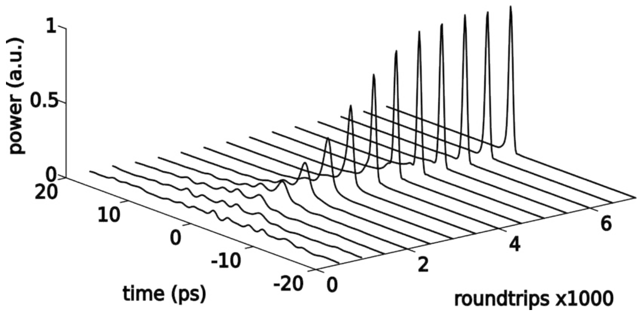

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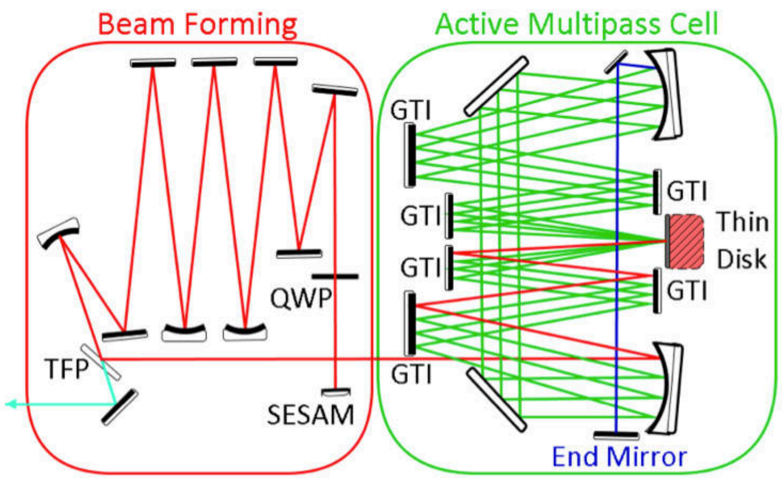
Helium flooding	45 W, 11 µJ, 790 fs
Multiple passes	145 W, 41 µJ, 1.1 ps



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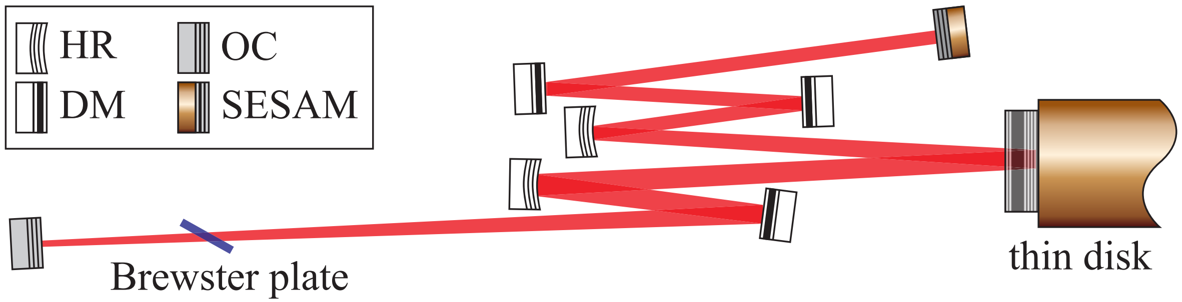

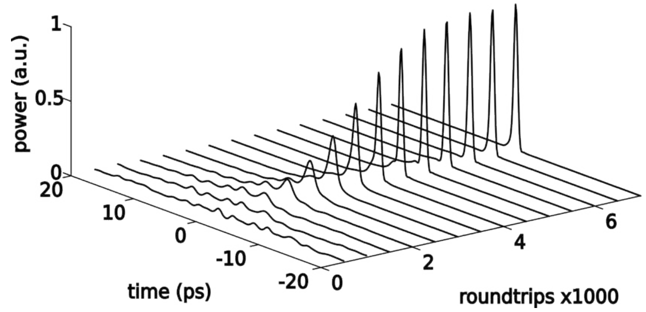

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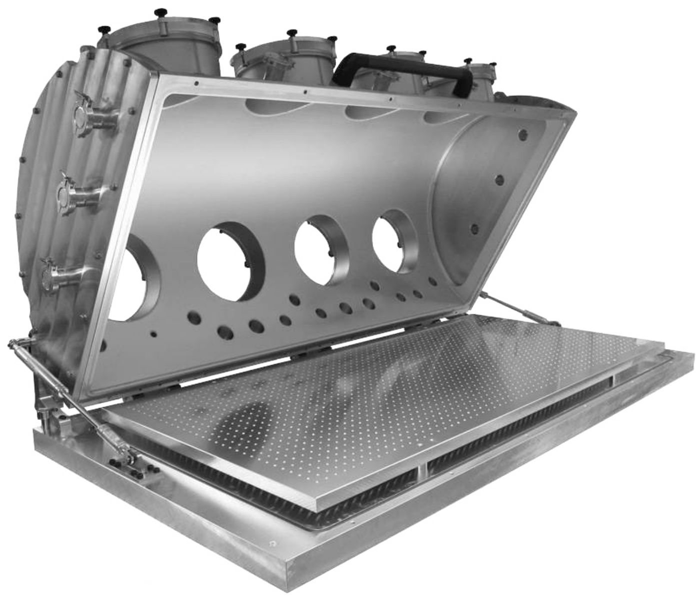
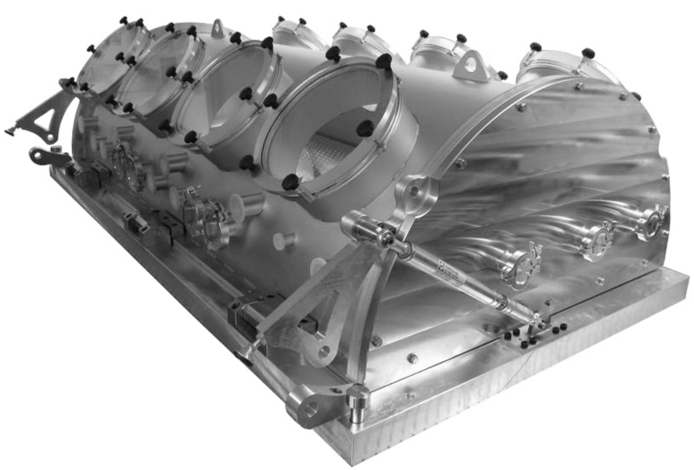
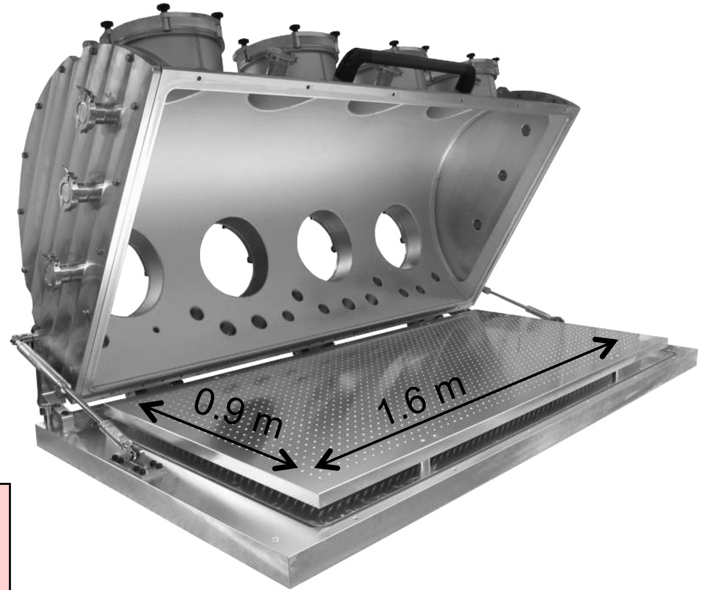
Helium flooding	45 W, 11 µJ, 790 fs
Multiple passes	145 W, 41 µJ, 1.1 ps
Vacuum	275 W, 17 µJ, 580 fs

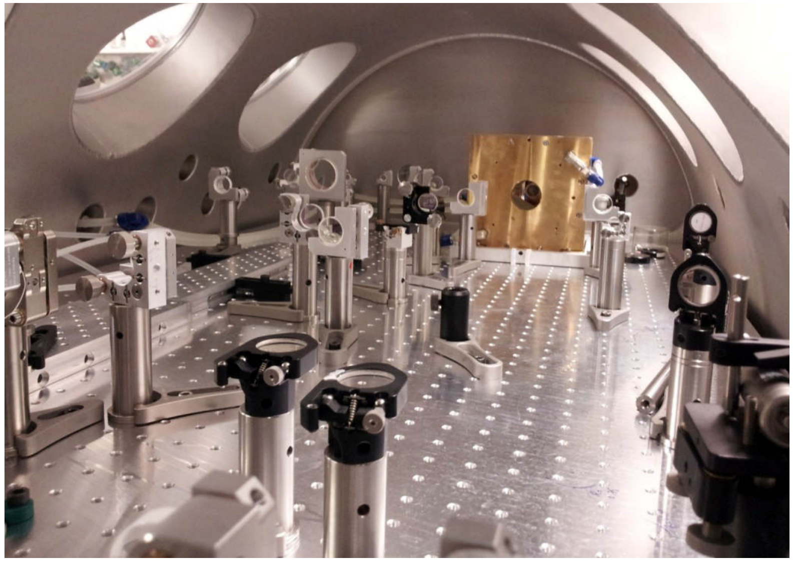


Vacuum environment

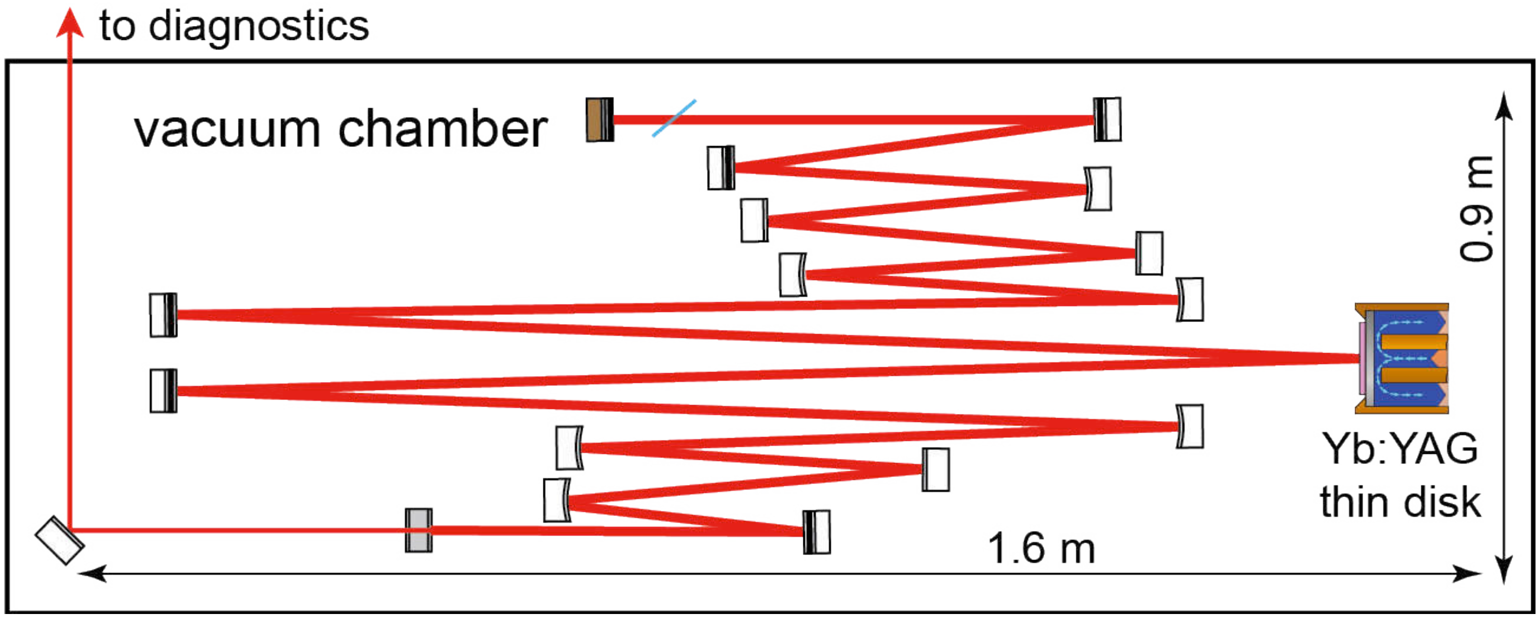




- ✓ minimum SPM
  - → small amount of dispersion required
- ✓ higher intracavity powers can be tolerated
  - → simple oscillator geometries with low number of passes
- ✓ easy adjustment of SPM by changing air pressure
- ✓ minimum pointing instabilities
- ✓ clean environment



Experimental setup



Disk (TRUMPF GmbH):

– < 100 µm thick, glued on water cooled diamond

Output coupling: 11.4%

Pump:

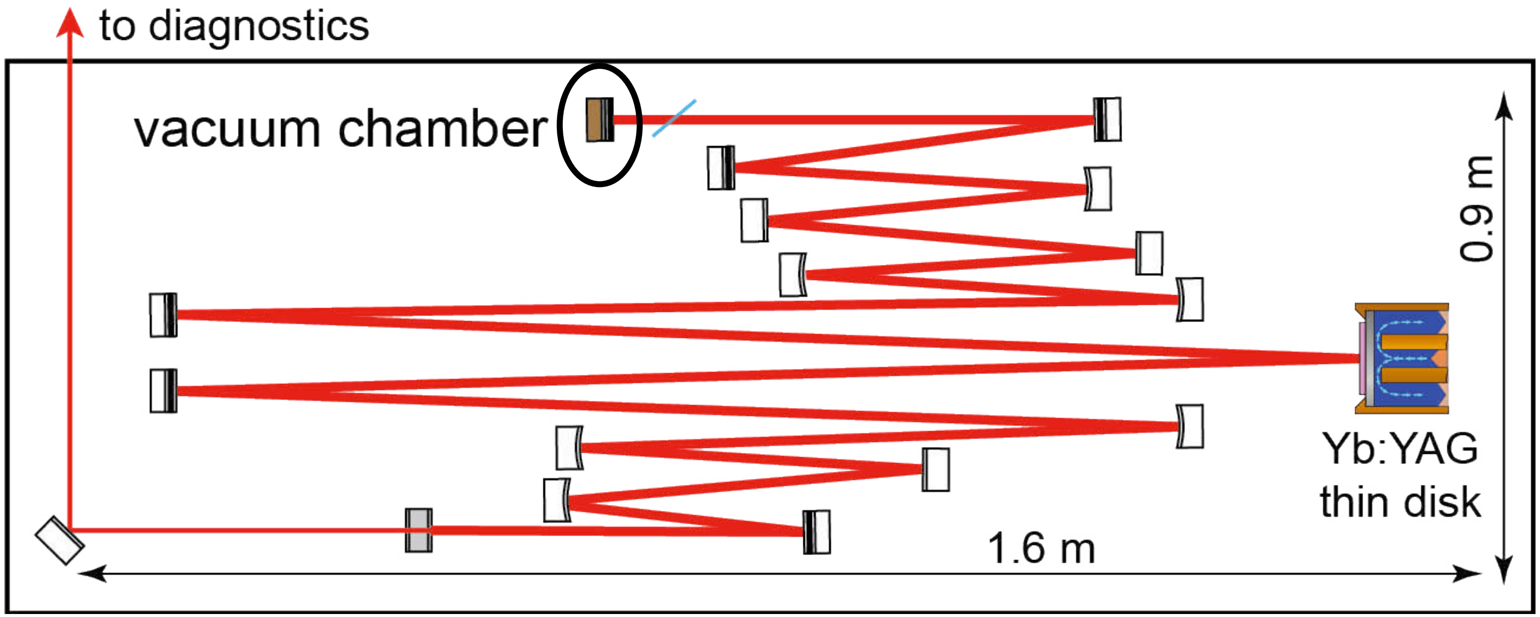
– 4.7 mm diameter, 24 passes

– λ_0= 940 nm, 1.2 kW available

Soliton modelocking:

- vacuum chamber pressure: 0.5 mbar
- 0.7 mm thick FS plate: polarization control
- total negative dispersion per roundtrip -8100 fs^2

Experimental setup

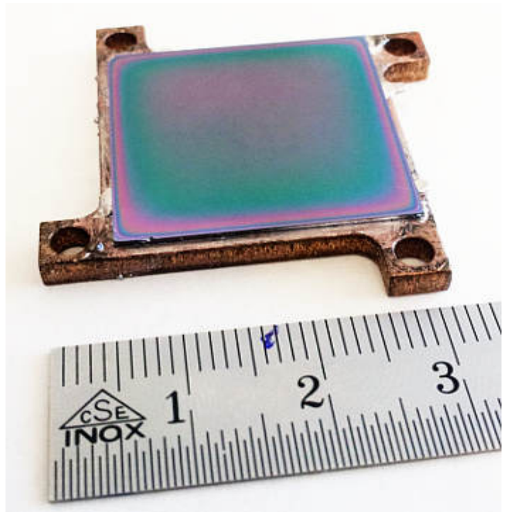


<u>SESAM with multiple QW and dielectric topcoating for high damage threshold</u>[#1]

– distributed Bragg reflector

– 3 QW: initially large ΔR

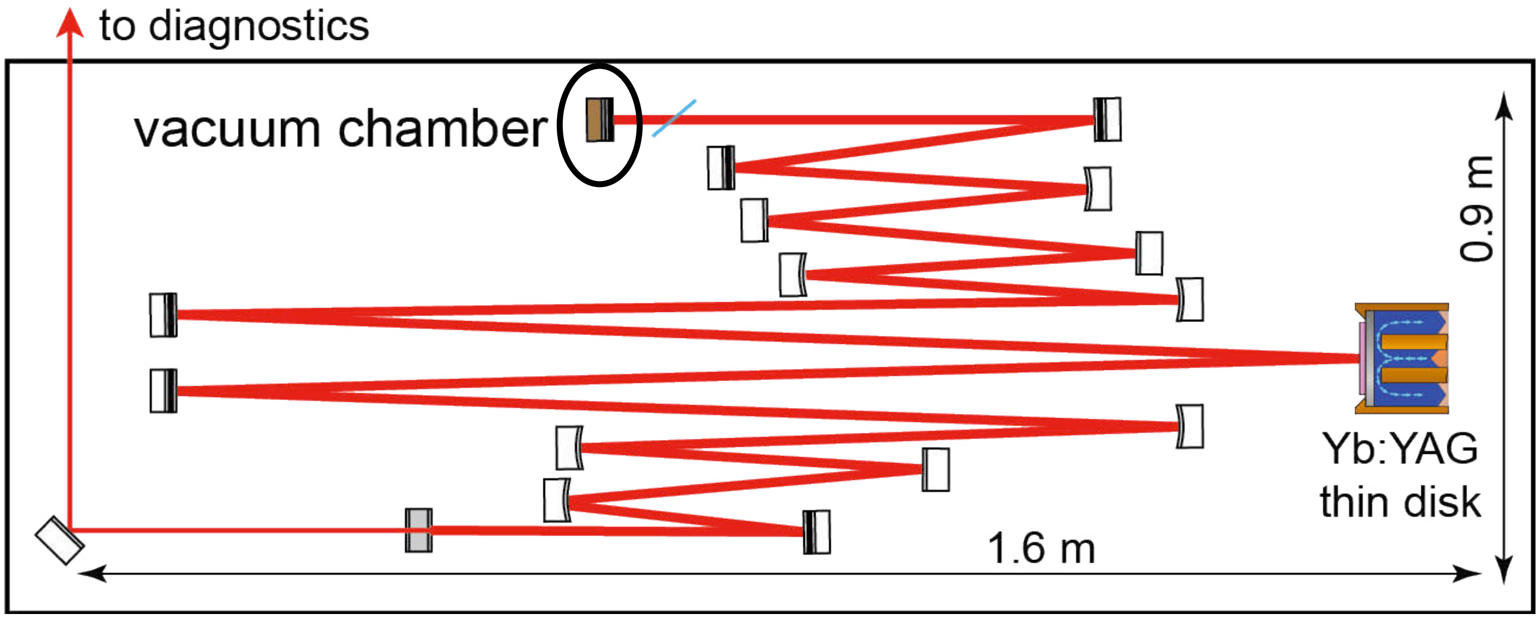
– 3 quarter-wave pair dielectric topcoating (PECVD) SiO_2/Si_3N_4



FIRST
Center for Micro- and Nanoscience

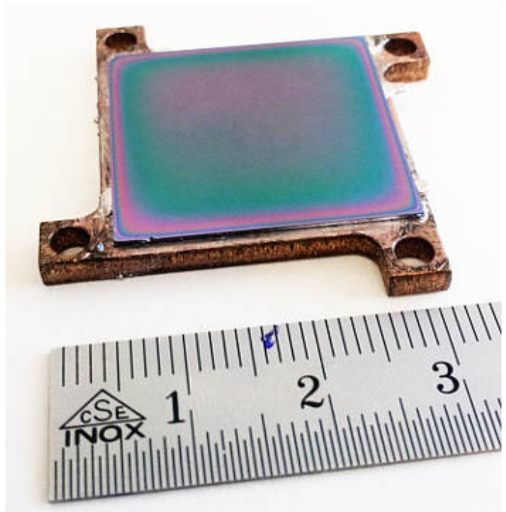
[#1] C.J. Saraceno, et al., *IEEE JSTQE*, vol 18, no.1, pp 29-41 (2012)

Experimental setup



SESAM with multiple QW and dielectric topcoating for high damage threshold[#1]

- F_{sat} = 140 µJ/cm^2
- ΔR = 0.95 %
- ΔR_{ns} = 0.1 %
- $\tau_{1/e}$ = 67 ps



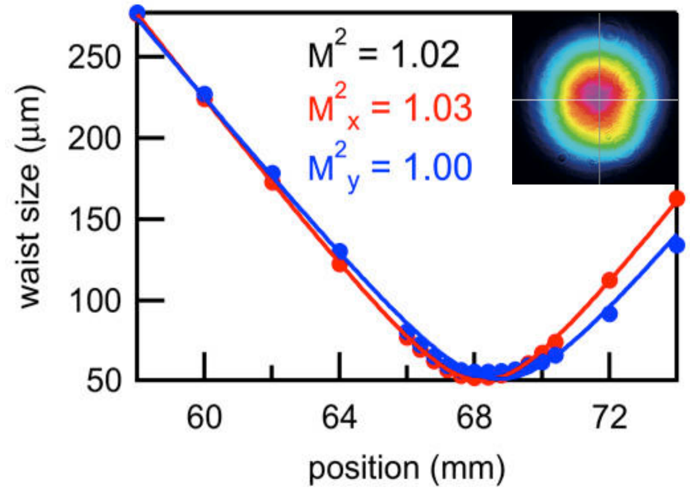
FIRST
Center for Micro- and Nanoscience

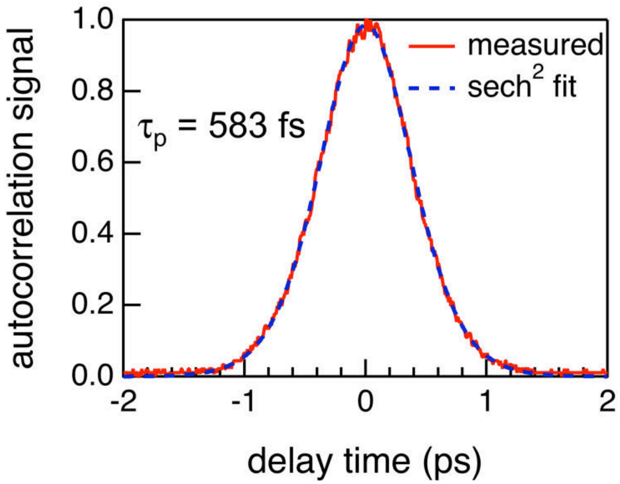
[#1] C.J. Saraceno, et al., *IEEE JSTQE*, vol 18, no.1, pp 29-41 (2012)

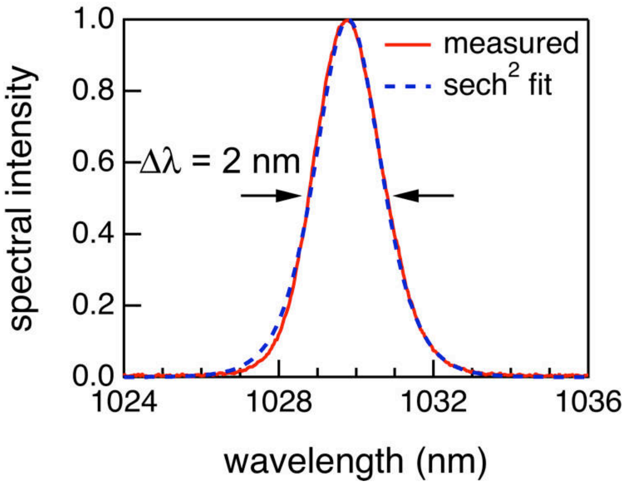
Results

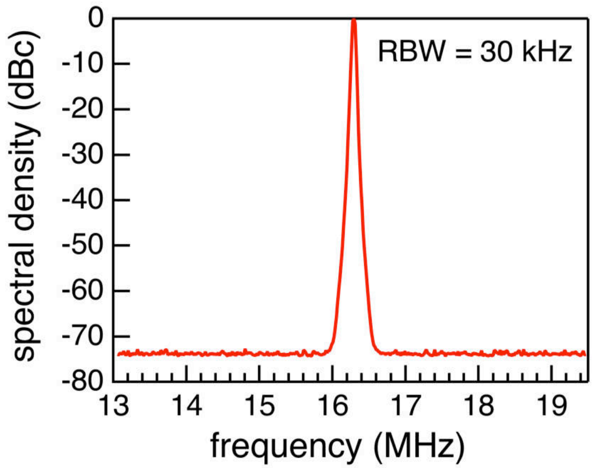
Result

P_{avg}	= **275 W**	τ_p	= 583 fs
P_{pump}	= 840 W	η_{opt}	= 32.4 %
f_{rep}	= 16.3 MHz	M^2	< 1.05
E_P	= 16.9 µJ	$\tau_P \cdot \Delta\nu$ = 0.329	
P_{pk}	= 25.6 MW	(ideal: 0.315)	









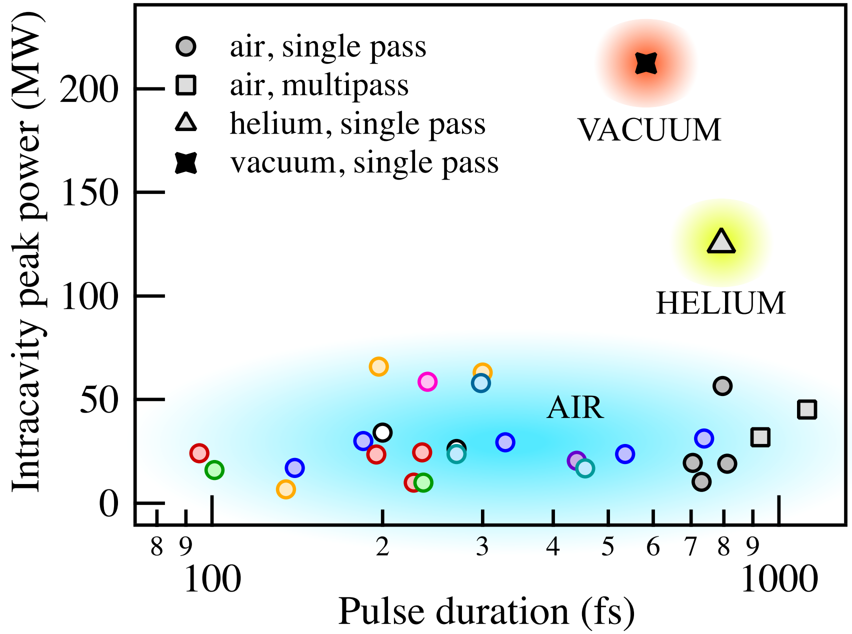
→ Highest average power from a passively modelocked oscillator
→ Main limitation to higher power: thermal effects on dispersive mirrors

[#1] C.J. Saraceno, et al., *Optics Express*, Vol. 20, No. 21, p. 23535 (2012)

Results

Result

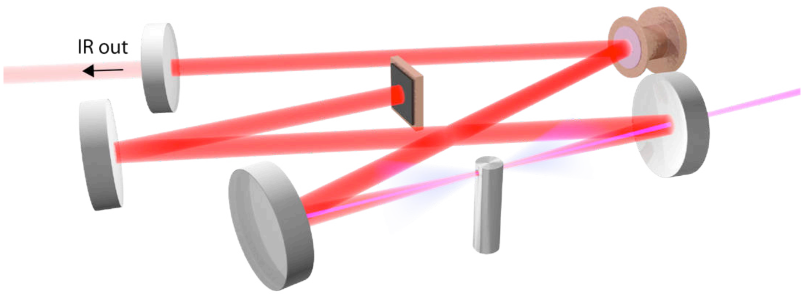
P_{avg}	=	**275 W**	τ_p	=	583 fs
P_{pump}	=	840 W	η_{opt}	=	32.4 %
f_{rep}	=	16.3 MHz	M^2	<	1.05
E_P	=	16.9 µJ	$\tau_P \cdot \Delta\nu$	=	0.329
P_{pk}	=	25.6 MW			(ideal: 0.315)



Intracavity

$P_{avg,ic}$	=	2.5 kW	✓	$\phi_{nl,vac}$ = 75 mrad
$E_{p,ic}$	=	154 µJ	✗	$\phi_{nl,air}$ = 1.64 rad
$P_{pk,ic}$	=	**236 MW**		

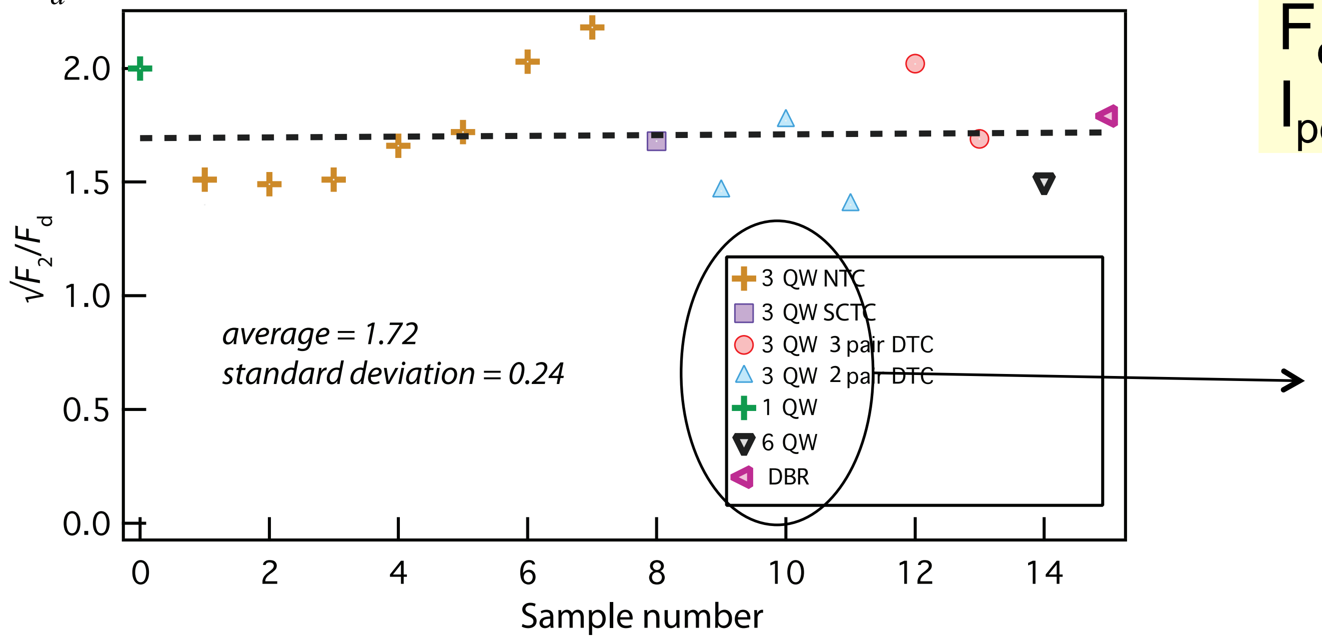
Intracavity nonlinear optics (HHG, THz, ...)



→ Vacuum chamber: essential

SESAM damage is not a problem

$\frac{\sqrt{F_2}}{F_d}$ **for 16 different samples**



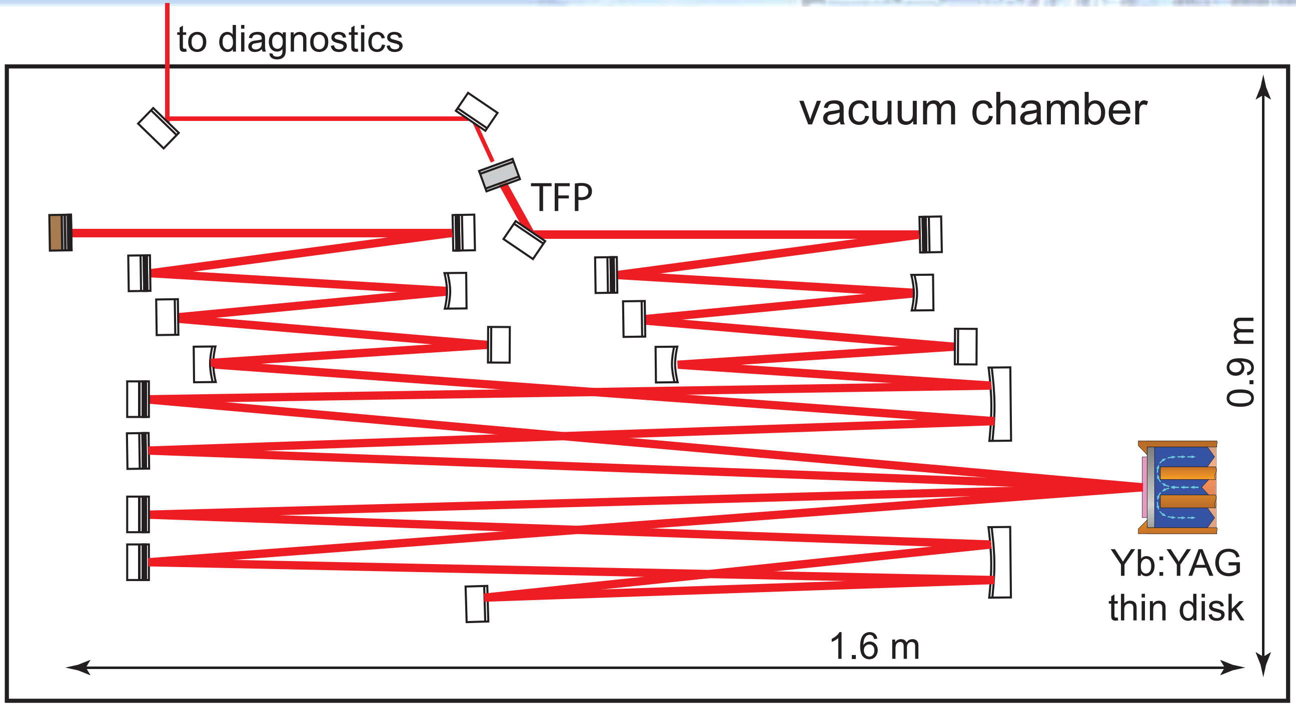
Best SESAMs:
F_d > 0.2 J/cm^2
I_{peak} > 370 GW/cm^2

Different absorbers :
DBR
1 QW
3 QW
6 QW

- **Damage threshold F_d scales with $\sqrt{F_2}$** independently of the number of QW
- Simple rule of thumb for SESAMs with increased damage threshold:
  - **minimal amount of GaAs layers**
  - **multiple quantum wells** and **dielectric topcoatings**

C. J. Saraceno et al., "SESAMs for high-power oscillators: design guidelines and damage thresholds", *IEEE JSTQE* **18**, 29, 2012 (online since Feb. 2010)

Towards higher energies



10 MHz cavity with two-passes through the disk:

- ✓ Higher output coupling rate
- ✓ Lower intracavity power

Disk (TRUMPF GmbH):

– < 100 µm thick, glued on water cooled diamond

Output coupling: 20.2%

Pump:

– 4.7 mm diameter, 24 passes

– λ_0= 940 nm, 1.2 kW available

Soliton modelocking:

– vacuum chamber pressure: 0.5 mbar

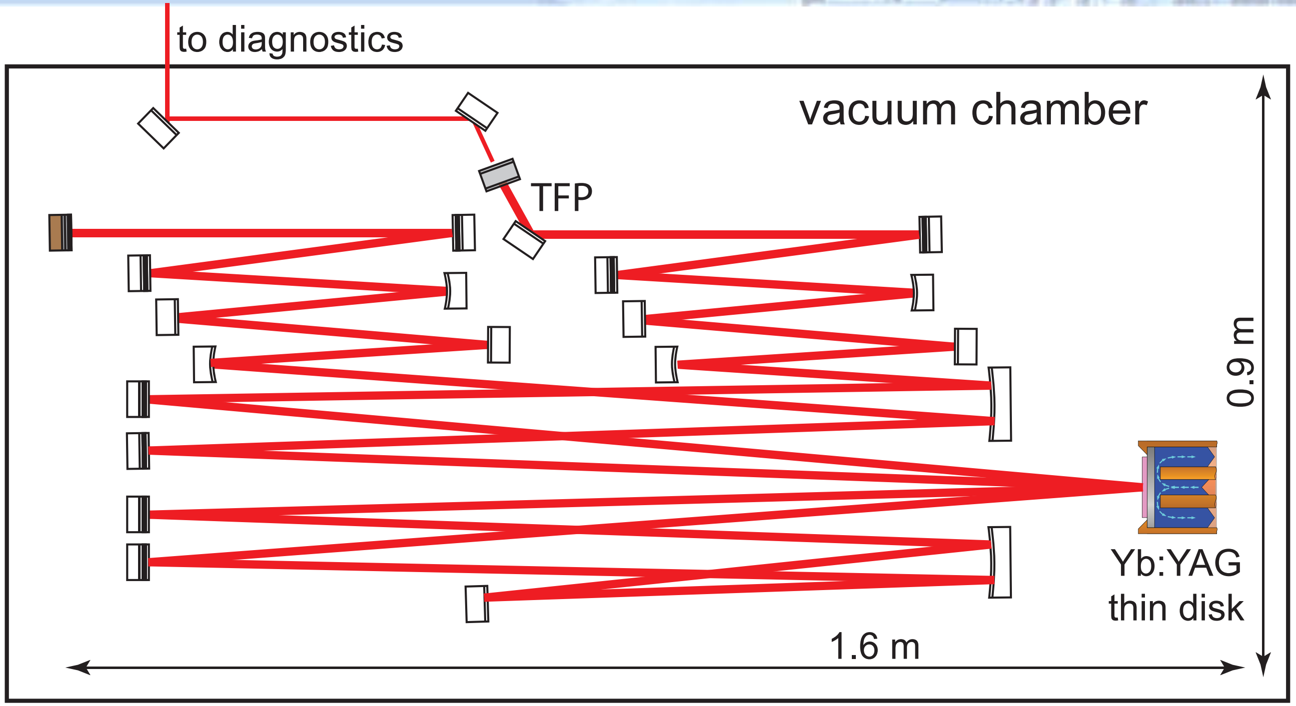
– **Thin film polarizer**

SESAM

– **Lower saturation fluence: 85 µJ/cm²**

– **Higher modulation depth: 1.5%**

Towards higher energies



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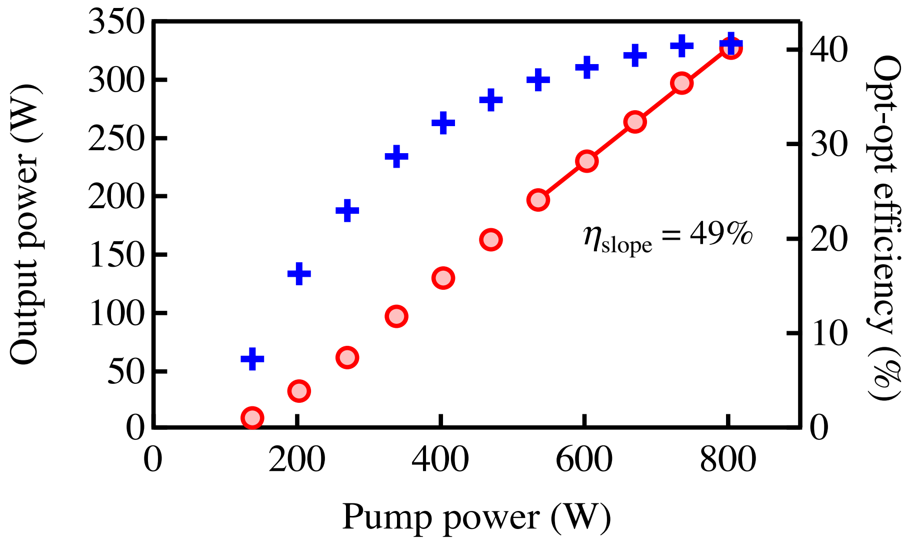
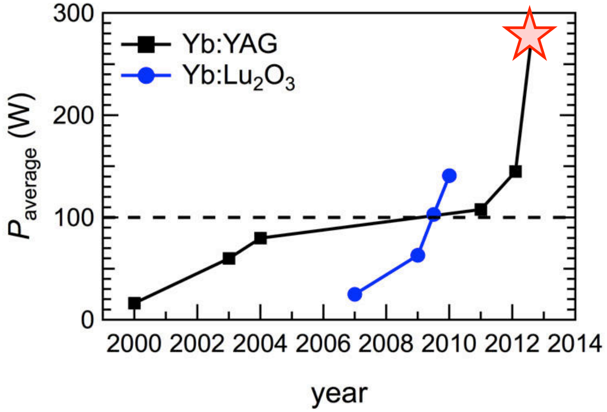


- ✓ CW fundamental mode: 330 W
- → Next step: energy scaling with passive multipass cell

Conclusions & Outlook

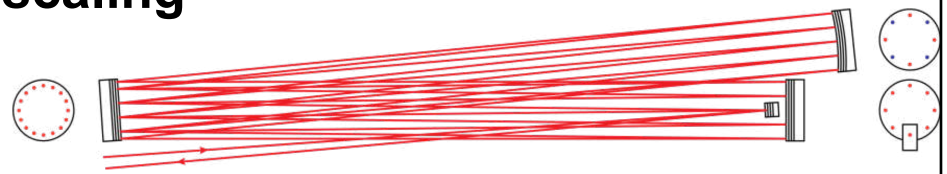
Conclusion: new approach for power scaling of fs high power TDLs

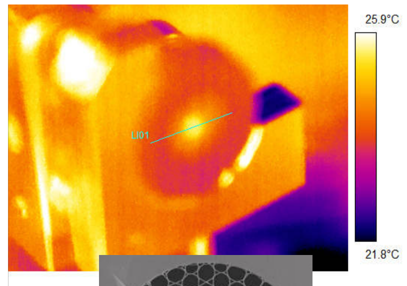
- Record high P_{out} from modelocked oscillator — P_{out} = 275 W
- Short pulses — τ_p = 583 fs
- High peak power — P_{pk} = 26 MW
- High pulse energy — E_P = 17 µJ

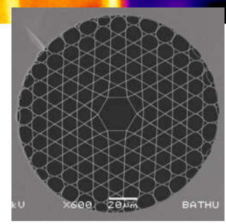


Outlook: further power and energy scaling

- Extend cavity length (Herriott-type cell)
  → increase pulse energy
- Dispersive mirrors with better thermal management
  → higher power at better optical-to-optical efficiency
- Further pump spot scaling
  → increase average power
- Pulse compression to sub-50 fs
  → increase peak power





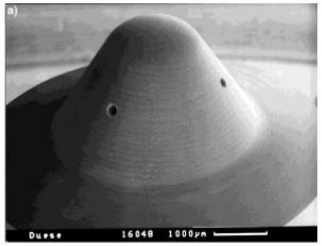


F. Emaury et al, We2.7 (12:00)

Applications

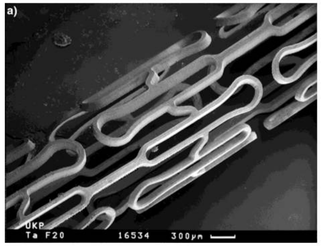
Industry

High-precision and high-speed micromachining



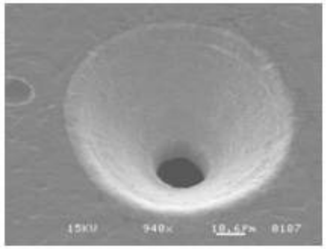
Fuel injection nozzles

Profeta et al., *Industrial Laser Solutions*, 2004



Stents

Nolte et al., *Adv. Eng. Mater.* 2, 2000



Inkjet nozzles

Liu et al., *Proc. SPIE*, Vol. 5713, 2005

Required

- High E_P > 10 µJ
- High P_{pk} > 10 MW

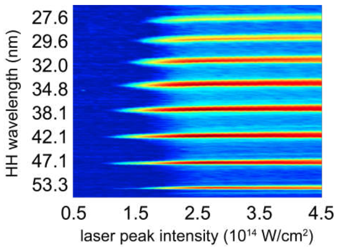
Wanted

- High f_{rep} (MHz)

➡ **High average power**
$\boldsymbol{P_{av} = E_p \cdot f_{rep}}$

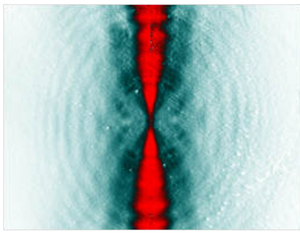
Science

Strong-field physics applications



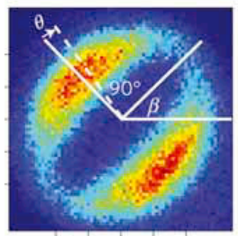
HHG

T. Auguste et al., *PRA* **80**, 033817 (2009)



Spectroscopy

T. Südmeyer et al., *Nat. Phot.* **2**, 599 (2008)



Attosecond science

A. Pfeiffer et al., *Nat. Phys.* **8**, 76 (2012)

Required

- High I_{pk} > 10^{14} W/cm^2
- Short pulses τ_p < 100 fs

Wanted

- High f_{rep} (MHz)

➡ **High average power**
$\boldsymbol{P_{av} = E_p \cdot f_{rep}}$